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Deflection estimation of two-way edge-supported slabs

Tahsin Reza Hossain and Md. Ruhul Alam[1]

Department of Civil Engineering
Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh

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Abstract

In designing a structure, members are so proportioned that will have adequate strength against failure and at the same time must posses sufficient stiffness to ensure serviceability. ACI Code provides minimum thickness of slabs so that the deflections are not excessive. It also allows thinner slabs if calculated deflections are found tolerable. The method given in ACI Code for deflection estimation uses Branson's equation to take into account cracking for short-term deflection calculation. As for long-term deflection, a simplified multiplier approach is proposed in the Code. Nonlinear finite element analysis based on the ACI method has been found [Alam (2003), Hossain and Alam (2003)] to give good correlation with experimental deflections. However, the method is not particularly suitable for designers, as it requires rigorous analysis, which is time-consuming and complicated. An attempt has been made in this paper to produce simplified design charts to estimate immediate deflection for different end conditions and aspect ratio. These charts have been found to produce realistic estimation of short-term deflection similar to finite element analysis as well as experimental results. Procedure for estimating long-term deflections has also been demonstrated with example.

Keywords: Concrete slab, deflection, slab thickness, serviceability.

1. Introduction

In strength design, the members are so proportioned that will have a proper safety margin against failure under a certain statistically obtained overload state. It is also important that the member performance in normal service be satisfactory. This performance, termed as serviceability, is not guaranteed simply by providing adequate strength. Service load deflections under full load may be excessively large, or long-term deflections due to sustained load may cause damage to partition walls. There are other serviceability related problems like visually disturbing wide tension cracks, vibrations causing discomfort etc.

[1] Graduate student

In the past, questions of serviceability were dealt with indirectly, by limiting the stresses in concrete and steel at service loads to rather conservative values that had resulted in satisfactory performance. Now, with strength design methods in general use that permit more slender members through more accurate assessment of capacity, with higher strength materials further contributing to the trend toward smaller member sizes, such indirect methods will no longer do. ACI Code (2002) proposes minimum slab thickness to ensure serviceability and at the same time allows thinner slabs if deflection calculation permits so.

ACI Code also provides a deflection calculation procedure based on Branson's (1977) equation. Hossain (1999), Hossain and Alam (2003) showed that ACI method predicts reasonable values of deflection. They also pointed that under normal loading conditions ACI minimum thickness guidelines are safe to use to ensure serviceability requirements. However, for excessive live load and larger panels, providing ACI Code minimum thickness may not be adequate. Also, for shorter spans with lighter loads, a smaller thickness may suffice from serviceability point of view. In both situations, deflection estimation is important in deciding a different thickness other than ACI thickness. However, deflection calculation using Finite Element Method incorporating effects of cracking, creep and shrinkage is difficult and time-consuming and not particularly suitable for the designers. In the current paper an attempt has been made to produce simplified design charts to predict deflection to help the designer selecting thickness other than ACI specified value.

2. Description of the model

A program module based on global plate stiffness approach has been developed by Hossain (1999) to incorporate the different short- and long-term models for predicting deflection of reinforced concrete slabs. The module acts as an integral part of the FE package FE77 (1999) and calculates modified elastic properties to represent cracking, creep and shrinkage for each element, on the basis of stresses of FE solution, which are then fed back into the assembly module of the FE package. Hossain and Vollum (2002), Vollum and Hossain (2002) and Vollum et.al. (2002) found good correlation in analysis of the real full scale 7 storied building at Cardington using this FE module employing EC2 (1992) and CEB-FIP Model Code 1990 MC-90 (1990) where creep and shrinkage deflections are dealt with more rigorously along with the effect of construction load. Deflection estimation procedure in ACI Code is simpler than these codes where long-term deflections are calculated from instantaneous deflection using multiplier. Branson’s crack model (1977) which is also adopted in the ACI Code (2002) to calculate instantaneous deflection has been used in the current work along with multiplier approach for long-term deflection.

Within the FE program, elastic moments in two principal directions for each element are calculated in the first run which are then used to calculate the effective moment of inertia in two principal directions using Branson's (1977) equation:

$$I_{e1} = \left(\frac{M_{cr}}{M_1}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_1}\right)^3\right] I_{cr1} \tag{1}$$

$$I_{e2} = \left(\frac{M_{cr}}{M_2}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_2}\right)^3\right] I_{cr2} \tag{2}$$

where, I_g and I_{cr} are gross and cracked moment of inertia of slab element. M_{cr} is the moment at which cracks occur and M_1 and M_2 are the principal moments.

Modification factors α_n and α_t for major and minor principal directions are calculated using:

$$\alpha_n = \frac{I_{e1}}{I_g} \tag{3}$$

$$\alpha_t = \frac{I_{e2}}{I_g} \tag{4}$$

The constitutive matrix [E′] is modified in the principal directions as follows for each element:

$$[E'] = \begin{bmatrix} \frac{\alpha_n E_c}{(1-\nu^2\alpha_n\alpha_t)} & \frac{\nu\alpha_n\alpha_t E_c}{(1-\nu^2\alpha_n\alpha_t)} & 0 \\ \frac{\nu\alpha_n\alpha_t E_c}{(1-\nu^2\alpha_n\alpha_t)} & \frac{\alpha_t E_c}{(1-\nu^2\alpha_n\alpha_t)} & 0 \\ 0 & 0 & \frac{E_c\sqrt{\alpha_n\alpha_t}}{2(1+\nu\sqrt{\alpha_n\alpha_t})} \end{bmatrix} \tag{5}$$

This [E′] matrix for each element is then transformed into global directions and fed back into the assembly module of the FE package. The analysis is repeated with the modified stiffness and the deflections obtained are therefore deflections considering cracking and tension stiffening. This Branson/ACI model has been found to give good prediction of experimental deflections, which are reported in Hossain (1999), Hossain and Alam (2003) etc. However, the method is not particularly suitable for the designers, as the rigorous FE analysis is time-consuming and not straightforward.

3. Simplified design charts

In order to facilitate the designer with simplified charts to estimate deflection, a large number of analysis have been carried out. It has been shown by Alam (2003) that for identical boundary conditions and aspect ratio, the ratio by which the immediate deflection increases from elastic deflection is reasonably same for different slabs and mostly depends on the level of cracking. To identify the level of cracking and increase in deflection due to cracking, the following terms have been introduced.

Stress ratio

Stress ratio is defined as the ratio between the maximum stress developed in the slab and modulus of rupture of concrete:

$$Stress\ \ ratio = \frac{Maximum\ \ \ developed\ \ \ stress}{Modulus\ \ of\ \ rupture\ \ of\ \ concrete} \tag{6}$$

Deflection ratio

The amount by which immediate deflection is increased from elastic deflection is termed as deflection ratio and defined as:

$$Deflection\ \ ratio = \frac{immediate\ deflection}{elastic\ deflection} \tag{7}$$

When the slab is not cracked, immediate deflection can be assumed to be equal to the elastic deflection considering gross cross-section and ignoring reinforcement.

3.1 Deflection ratio-Stress ratio curve

Nine sets of design charts have been developed for each boundary condition case (shown in Fig. 1) with a modulus of rupture of $0.33\sqrt{f_c'}$ (MPa), which are presented in Figs. 2 to 10. A large number of FE analyses has been performed where level of cracking has been varied by changing load and deflection-ratios are plotted against stress-ratios. In each slab case, separate curves for different aspect ratios have been found and formed a band. For slab case 1, the upper curve corresponds to aspect ratio 1.00 and lower one to 0.50 and for rest of the slab cases 2 to 9, a reverse trend has been observed. It has been shown by Alam (2003) that deflection ratio stress ratio curves are only influenced by level of cracking, boundary condition and aspect ratio of slab.

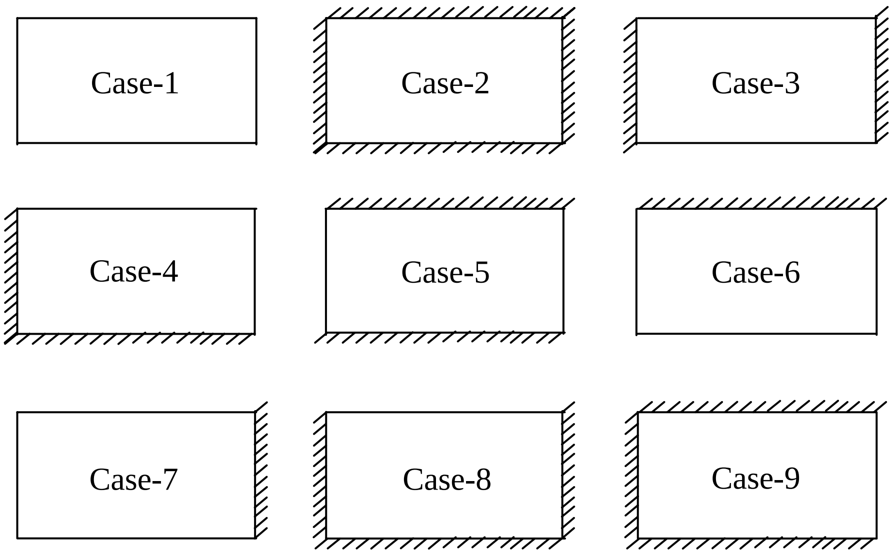

Fig. 1 Edge-supported slabs with different end conditions

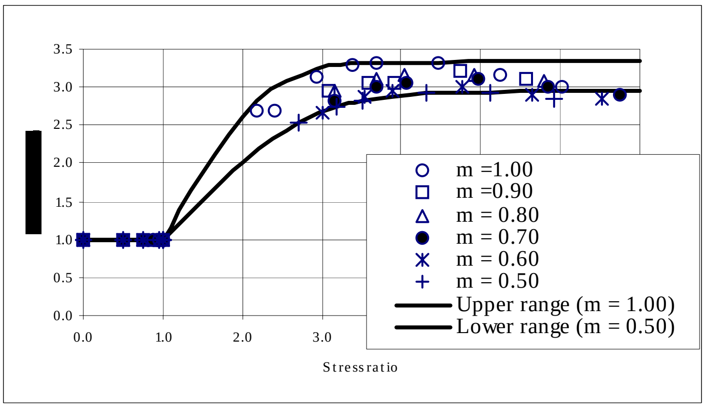

Fig. 2 Deflection ratio vs. stress ratio chart of edge supported slab for case-1, $f_r = 0.33\sqrt{f_c'}$ MPa

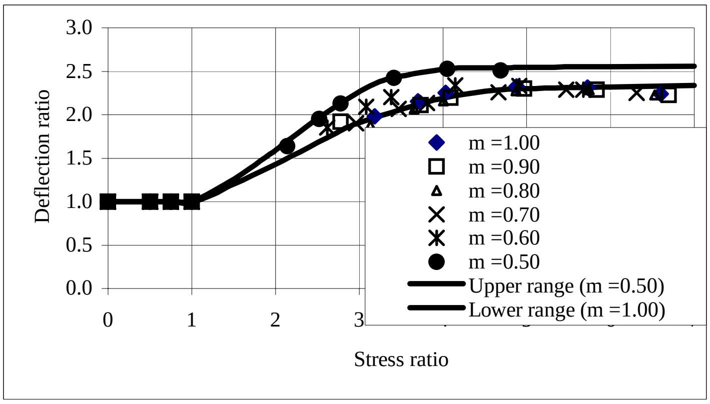

Fig. 3 Deflection ratio vs. stress ratio chart of edge supported slab for case-2, $f_r = 0.33\sqrt{f'_c}$ MPa

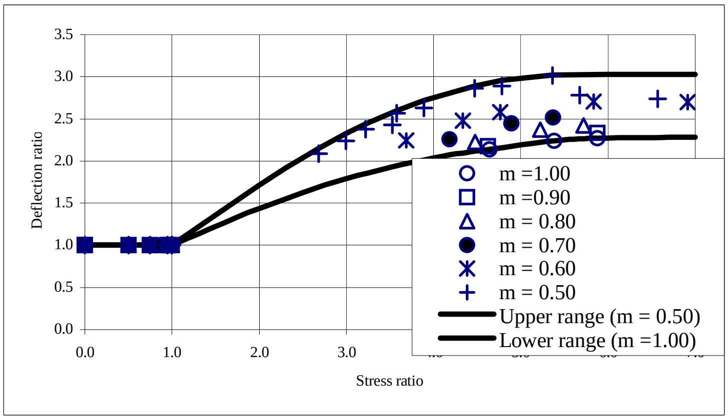

Fig. 4 Deflection ratio vs. stress ratio chart of edge supported slab for case-3, $f_r = 0.33\sqrt{f'_c}$ MPa

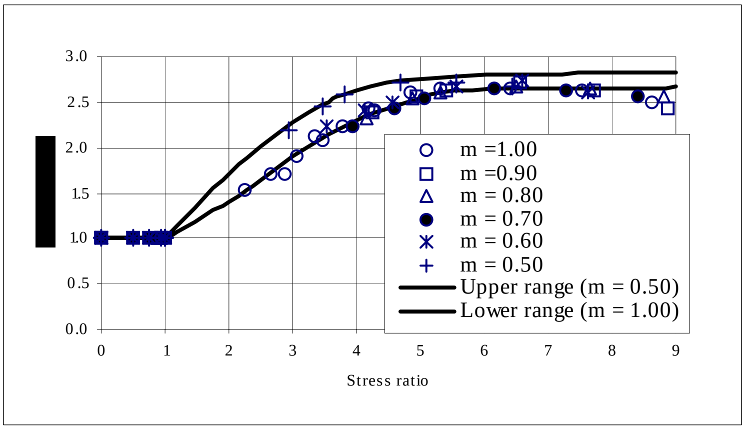

Fig. 5. Deflection ratio vs. stress ratio chart of edge supported slab for case-4, $f_r = 0.33\sqrt{f'_c}$ MPa

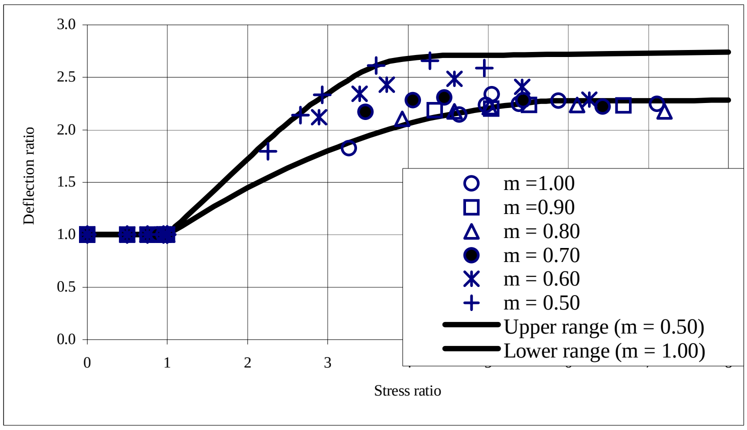

Fig. 6 Deflection ratio vs. stress ratio chart of edge supported slab for case-5, $f_r = 0.33\sqrt{f'_c}$ MPa

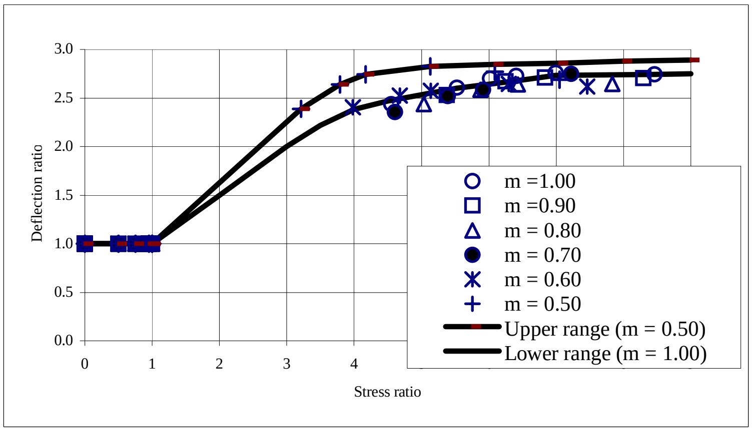

Fig. 7 Deflection ratio vs. stress ratio chart of edge supported slab for case-6, $f_r = 0.33\sqrt{f'_c}$ MPa

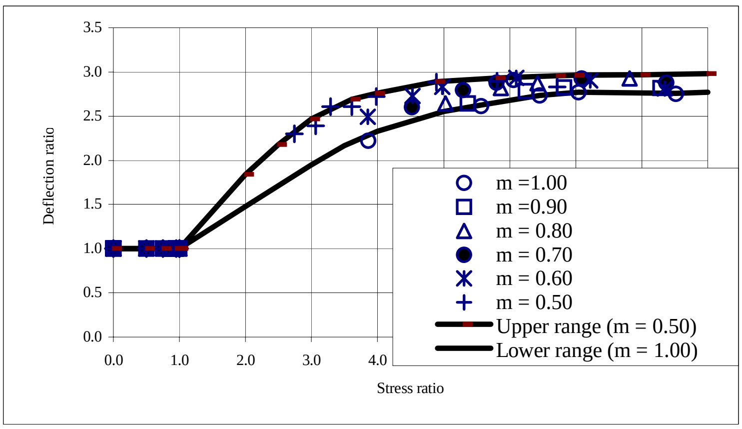

Fig. 8 Deflection ratio vs. stress ratio chart of edge supported slab for case-7, $f_r = 0.33\sqrt{f'_c}$ MPa

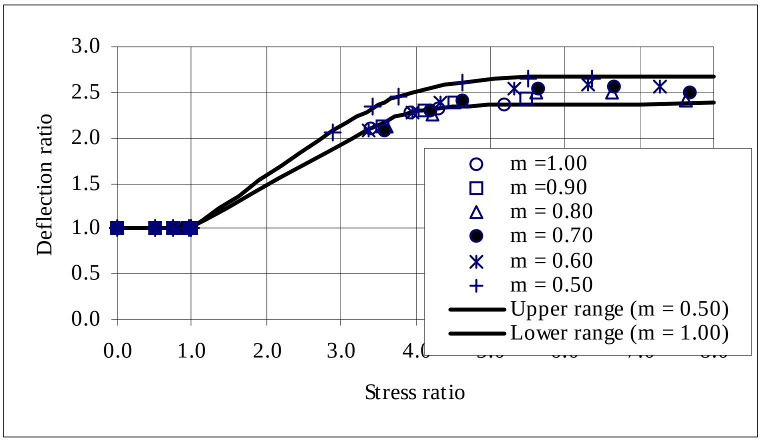

Fig. 9 Deflection ratio vs. stress ratio chart of edge supported slab for case-8, $f_r = 0.33\sqrt{f'_c}$ MPa

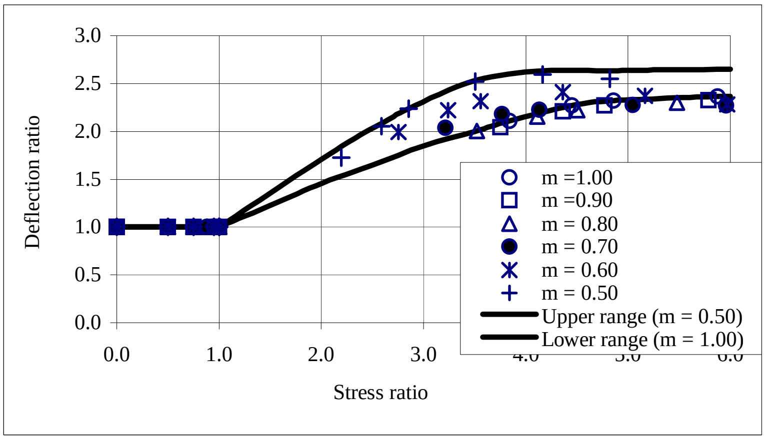

Fig. 10 Deflection ratio vs. stress ratio chart of edge supported slab for case-9, $f_r = 0.33\sqrt{f'_c}$ MPa

A designer can calculate the immediate deflection using these design charts. There are two other accompanying charts (presented in Tables 1 and 2) to calculate maximum developed stress and elastic deflections. A similar design chart has been presented in Ahmed and Chowdhury (1999) for calculation of elastic deflection. The stress ratio up to unity represents no cracking of slab and immediate deflection will be equal to elastic deflection. The stress ratio greater than unity represents the cracking of slab and immediate deflection will be greater than the elastic deflection and it can be calculated by multiplying the above mentioned deflection ratio to elastic deflection.

3.2 Moment coefficient for calculation of maximum developed stress

The bending moment coefficients of two-way edge-supported slab have been calculated from the finite element program FE-77. The coefficients predicted from FE-77 are presented in the Table 1 for calculation of maximum developed stress. For all the cases, except case-1, the negative moment coefficients at continuous edge in the shorter direction are presented in the table. For case 1, maximum moment occurs at midspan in shorter direction.

3.3 Calculation of elastic deflection of slab

Finite element analyses have been carried out to generate deflection coefficients for easy calculation of elastic deflection of two-way edge-supported slabs. The calculated deflection coefficients are presented in the Table 2 for different boundary conditions and aspect ratios. A designer can easily calculate the elastic deflection of slab using the following equation:

$$\delta = D_a \frac{w l_a^4}{E t^3} \tag{8}$$

where,

D_a = elastic deflection coefficient
w = total load applied on the slab
l_a = span length of slab in short direction
E = modulus of elasticity of concrete
t = thickness of slab

Table 1
Moment coefficients for calculation of immediate deflection of slab

Aspect ratio ($m = l_a/l_b$)	Case-1	Case-2	Case-3	Case-4	Case-5	Case-6	Case-7	Case-8	Case-9
	$C_a(+)$	$C_a(-)$	$C_b(-)$	$C_a(-)$	$C_a(-)$	$C_a(-)$	$C_b(-)$	$C_a(-)$	$C_a(-)$
1.00	0.0403	0.0439	0.064	0.0577	0.064	0.0762	0.0767	0.0468	0.0529
0.90	0.0483	0.0505	0.0588	0.0668	0.0681	0.0844	0.0683	0.0557	0.0589
0.80	0.0578	0.0572	0.0523	0.0761	0.0717	0.0919	0.0587	0.066	0.0643
0.70	0.0688	0.0646	0.0446	0.0856	0.0755	0.1002	0.0482	0.0783	0.07
0.60	0.081	0.0704	0.0357	0.0947	0.0778	0.107	0.0373	0.0904	0.0741
0.50	0.0941	0.0746	0.0261	0.1024	0.0787	0.112	0.0267	0.1012	0.0766

3.4 Calculation of immediate deflection

The calculation procedure of immediate deflection of two-way edge-supported reinforced concrete slabs for different edge conditions and aspect ratios are shown here. For a slab panel, the designer select a slab thickness. Using concrete properties and total applied load the elastic deflection can be calculated from Eq.(8).

The stress developed at support or midspan is then calculated for the selected slab thickness and total load:

$$f = \frac{6cwl_a^2}{bt^2} \tag{9}$$

where,

f = stress developed in the slab
b = width of slab usually taken as unity
c = moment coefficient calculated from FE analysis (shown in Table 1)

Table 2
Coefficient for elastic deflection of two-way edge-supported slab centre, considering short direction

Aspect Ratio ($m = l_a/l_b$)	Case-1 (10^{-2}) D_a	Case-2 (10^{-2}) D_a	Case-3 (10^{-2}) D_a	Case-4 (10^{-2}) D_a	Case-5 (10^{-2}) D_a	Case-6 (10^{-2}) D_a	Case-7 (10^{-2}) D_a	Case-8 (10^{-2}) D_a	Case-9 (10^{-2}) D_a
1.00	4.8332	1.4817	2.2542	2.5977	2.2542	3.3789	3.3789	1.8824	1.8824
0.95	5.3403	1.6346	2.6233	2.8652	2.3654	3.6324	3.8357	2.1365	2.0212
0.90	5.8970	1.7956	3.0585	3.1581	2.4741	3.8963	4.3553	2.4208	2.1615
0.85	6.5050	1.9635	3.5701	3.4673	2.5797	4.1679	4.9434	2.7368	2.3019
0.80	7.1650	2.1347	4.1703	3.7889	2.6800	4.4433	5.6053	3.0811	2.4367
0.75	7.8762	2.3052	4.8693	4.1256	2.7738	4.7184	6.3445	3.4545	2.5705
0.70	8.6363	2.4706	5.6761	4.4743	2.8584	4.9872	7.1617	3.8563	2.6947
0.65	9.4385	2.6265	6.5951	4.8013	2.9333	5.2471	8.0521	4.2573	2.8051
0.60	10.271	2.7673	7.6256	5.1483	2.9949	5.4992	9.0088	4.6809	2.9035
0.55	11.124	2.8870	8.7539	5.4484	3.0420	5.7371	10.013	5.0783	2.9800
0.50	11.971	2.9801	9.9548	5.7246	3.0752	5.9472	11.048	5.4410	3.0390

The stress ratio (f/f_r) is then calculated which signifies the level of cracking. The corresponding deflection ratio is then obtained from the deflection ratio-stress ratio curve for the slab. Finally the immediate deflection is calculated by multiplying the elastic deflection with deflection ratio:

$$\textit{Immediate deflection} = \textit{Deflection ratio} \times \textit{Elastic deflection} \tag{10}$$

A designer can easily calculate the immediate deflection of two-way edge-supported reinforced concrete slabs for different edge conditions and aspect ratios without help of any finite element program.

4. Validation of the FE analysis and the simplified charts

4.1 Comparison of Immediate Deflections computed from FE Analysis and estimated using design charts

For the purpose of validation of these deflection ratio stress curves, slabs with all possible end conditions have been considered with changing aspect ratio, applied loading and slab dimension. For any aspect ratio between 0.5 and 1.0, the respective deflection ratio has been calculated by interpolation. Deflections computed from FE analysis and estimated using design charts compare well, which are presented in Alam (2003).

4.2 Validation of the FE analysis and the simplified charts with experiments: Shukla slab

The FE model of simulating cracks and the developed design charts for prediction of immediate deflection of two-way slabs have been verified with the experimental results of slabs tested by Shukla & Mittal. For this purpose three slabs S-8, S-11 and S-12 have been considered. Shukla & Mittal (1976) carried out a series of tests on two-way edge-supported slabs. All the slabs were 214 cm square and 8 cm thick. The slabs were supported on reinforced concrete walls with centre to centre span of 183 cm each way. Their corners were held down by means of 40 mm diameter steel rods anchored to the floor. Loads were applied to the test slab in increments of 2 tonnes each through an inverted waffle-tree system which transferred load at 16 equidistant point of the slab. Three slabs (S-8, S-11 and S-12) from this series have been analysed here. S-8 and S-11 were isotropically reinforced with 10 mm bars to provide 5.24 and 4.36 cm^2/m steel in each direction respectively. S-12 is reinforced with 10mm and 6mm bars to make an orthotropic slab with 5.24 and 1.35 mm^2/m steel in two directions. The three slabs differ in concrete strengths which were 15.9 (S-8), 22.0 (S-11) and 19.2 (S-12) N/mm^2. Moduli of rupture and elasticity were not reported and hence have been estimated using the ACI equations. Details of the slab dimensions and FE mesh are shown in Fig. 10. The deflections calculated using design charts and FE analysis are presented in Figs. 11 to 13 along with experimental results.

5. Example showing short- and long-term deflections estimation

To demonstrate the method of deflection calculation following ACI Code, an example is worked out here. Unlike the approach shown in Nilson (1997), cracking in slab is considered in the analysis. In the current example, short- and long-term deflections of a 3.66 m x 4.27 m corner panel slab have been estimated with following parameters.

Slab thickness has been calculated using formulae given in ACI Code (2002) and found to be 95 mm (rounding to 100 mm is not done in this study), the following parameters are assumed: f_c'= 20.7 MPa, f_y= 414 MPa, E_c= 20.7 GPa, Poisson's ratio =0.18 and Modular ratio, n=10. A reduced value of $0.33\sqrt{f_c'}$ (MPa) has been used for rupture strength of concrete instead of $0.62\sqrt{f_c'}$ (MPa). Tam and Scanlon (1986) produced good correlation between calculated deflection with $0.33\sqrt{f_c'}$ value and mean field-measured deflection. This approach of using reduced modulus of rupture to take into the effect of cracking due to restraint shrinkage is reported in a series of papers [ACI Committee 435 (1991), Thompson & Scanlon (1988), Scanlon & Murray (1982), Ghali (1990)].

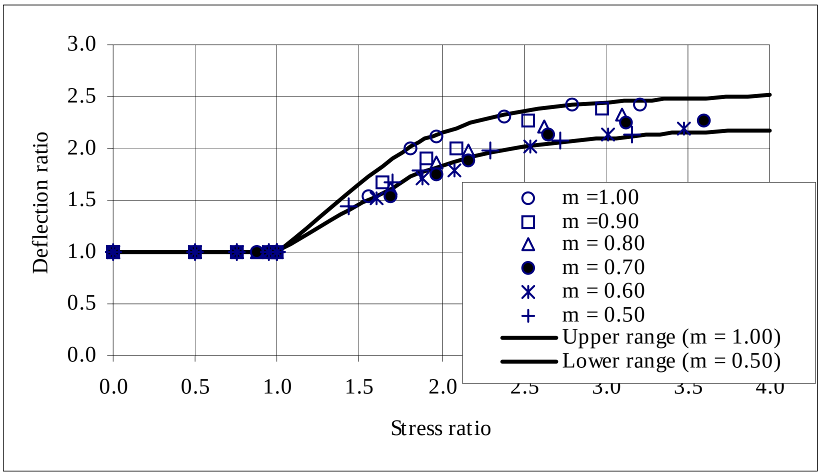

Fig. 11 Deflection ratio vs. stress ratio chart of edge supported slab for case-1, $f_r = 0.62\sqrt{f_c'}$ MPa

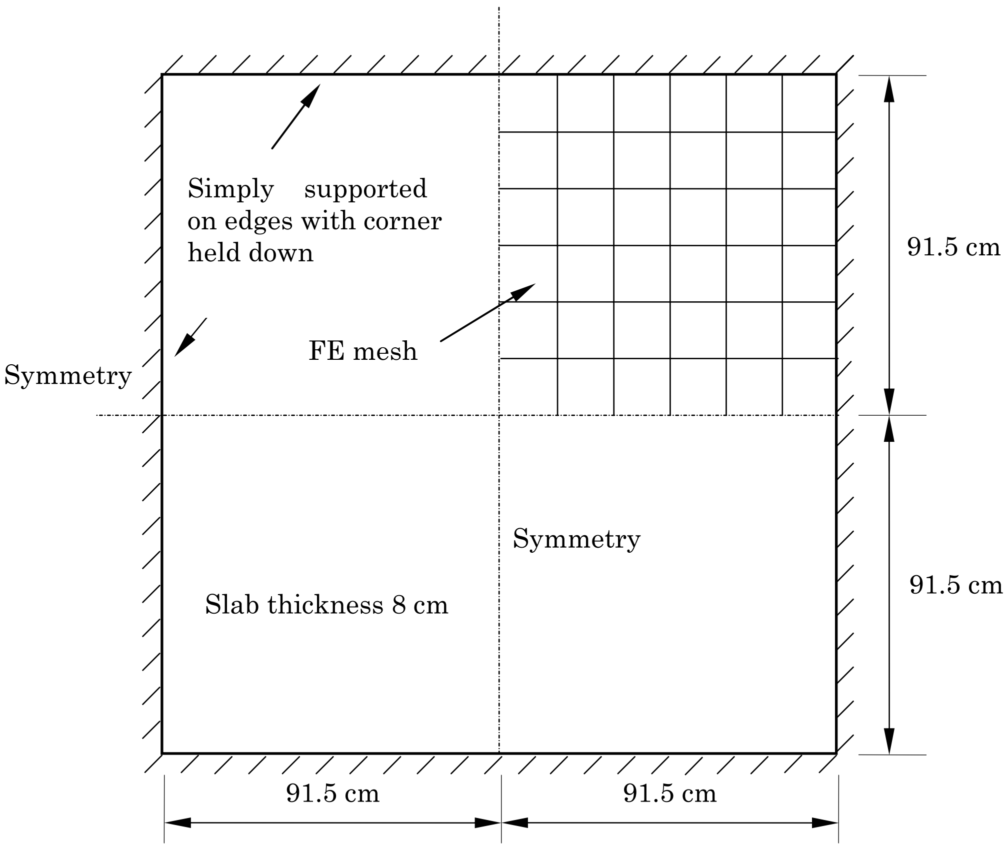

Fig. 12 Details of edge-supported two-way slabs tested by Shukla & Mittal (1976)

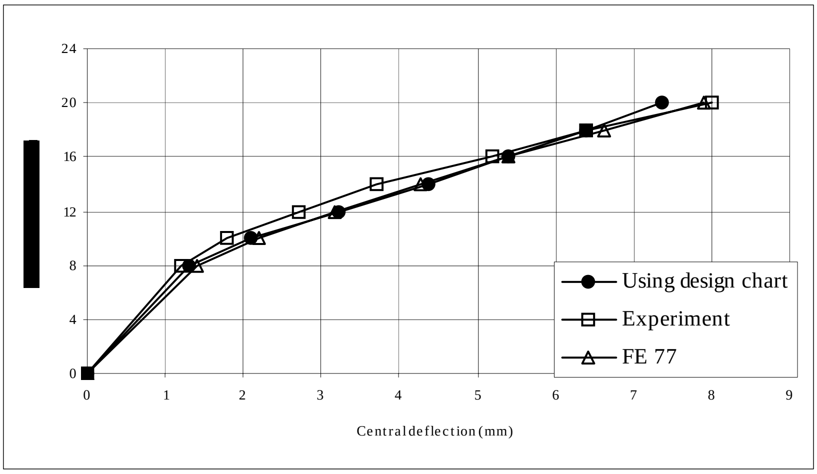

Fig. 13 Load-deflection curve for Shukla & Mittal slab, S-8.

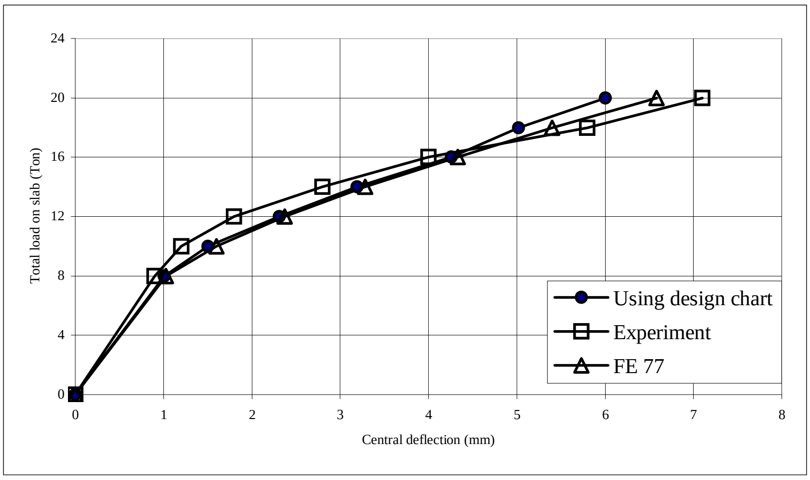

Fig. 14 Load-deflection curve for Shukla & Mittal slab, S-11.

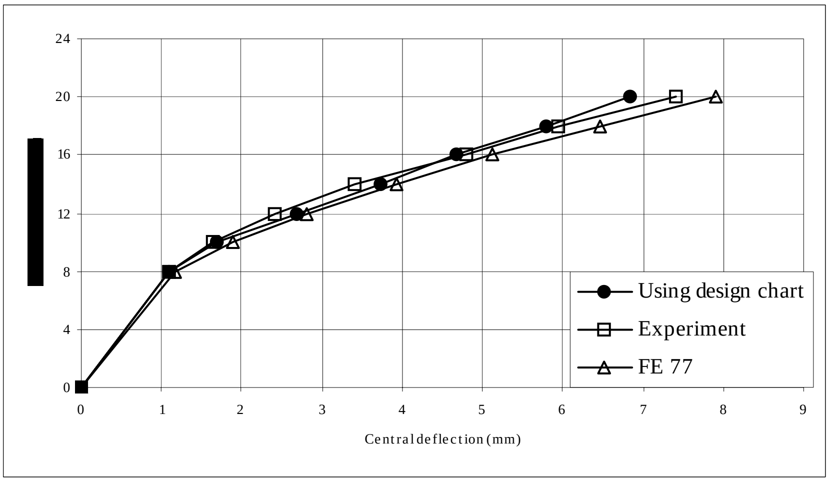

Fig. 15 Load-deflection curve for Shukla & Mittal slab, S-12.

The total dead load with 1.2 kN/m^2 of floor finish was 3.45 kN/m^2 and live load was 3.83 kN/m^2. Full panel has been modeled with 24 x 20= 480 plate elements and from FE analysis using ACI crack model, the immediate deflection has been found for total dead and live load.

On the basis of empirical studies, ACI Code (2002) specifies that additional long-term deflection due to combined effects of creep and shrinkage shall be calculated by multiplying the immediate deflection by the factor

$$\lambda = \frac{\xi}{1+50\rho'} \tag{11}$$

where,
ξ= a time-dependant coefficient, Branson has suggested a five-year value of ξ=3.0 for two-way slabs, $\rho' = A_s'/bd$, is usually zero for slabs as compression steels are seldom used.

The calculation of long-term deflection has been performed using sustained load of 30% live load and ξ =3.0 as proposed by Branson for slabs.

5.1 Estimation of immediate deflection

The aspect ratio of the panel, m = 0.857 and corresponding deflection coefficient and moment coefficient has been calculated from Table 1 and 2 respectively. The required values are:

Deflection coefficient, $D_a = 3.41\times 10^{-2}$
Moment coefficient, c = 0.0705
Span length, l_a = 3.66 m
Total load on slab, w = 7.28 kN/m^2 =7.28 X 10^{-3} N/mm^2
Modulus of rupture of concrete, f_r = 1.501 N/mm^2

Elastic deflection, $\delta = \frac{D_a w l_a^4}{E t^3}$

$$= \frac{3.41\times10^{-2} \times 7.28\times10^{-3}\times(3660)^4}{20.7\times10^3\times(95)^3}$$

= 2.51 mm

The flexural stress developed at support in the short direction,

$$f = \frac{6cwl_a^2}{bt^2}$$

$$= \frac{6\times 0.0705\times 7.28\times 10^{-}\times(3660)^2}{1\times(95)^2}$$

= 4.57 MPa

The stress ratio, f /f_r = 4.57/1.501 = 3.04
The deflection ratio predicted from design chart (Fig. 5) = 2.0
The immediate deflection = Elastic deflection × Deflection ratio
= 2.51 × 2
= 5.02 mm
From FE analysis of the slab the immediate deflection has been found = 4.57 mm
Percent variation with respect to FE analysis = 10 %

5.2 Long-term deflection calculation from design charts

The calculation of immediate deflection of a slab for different end conditions from design charts are discussed. With the help of those design chart the immediate deflection of the above slab has been estimated to be 5.02 mm. The calculation of incremental and total deflections from above procedure are presented as follows.

Immediate deflection for dead load and live load from design chart, Δ_{d+l} = 5.02 mm
The time-dependent portion of dead load deflection is,

$$\Delta_d = 5.02 \times \frac{3.45}{7.28} \times 3 = 7.14 \text{ mm}$$

The long-term deflection due to sustained portion of the live load is

$$\Delta_{0.3L} = 5.02 \times \frac{3.83}{7.28} \times 0.3 \times 4 = 3.17 \text{ mm}$$

The instantaneous deflection due to application of short-term portion of the live load is

$$\Delta_{0.7L} = 5.02 \times \frac{3.83}{7.28} \times 0.7 = 1.85 \text{ mm}$$

The total incremental deflection is Δ = 7.14 + 3.17 + 1.85 = 12.16 mm

The ACI Code limitation of incremental deflection is $\frac{l}{480}$ = 7.63 mm, it is observed that the slab thickness needs to be increased to control the incremental deflection of slab.

The total deflection is $\Delta_{total} = 7.14 \times \frac{4}{3} + 3.17 + 1.85$ = 14.54 mm

The ACI Code limitation of total deflection is $\frac{l}{240}$ = 15.25 mm. From calculation, slab thickness is found to be adequate regarding total deflection.

6. Conclusions

Hossain and Alam (2003) showed that in most cases the thickness provided by the ACI Code (2002) proved to be adequate where spans, live loads, concrete strength etc. are in normal range. However, for shorter spans with lighter loads, a smaller thickness may suffice from serviceability point of view. The ACI Code allows slab thickness less than the specified value if calculated values are within code-specified limits. So it would be economical to use thinner slabs in such situations where deflection analysis permits so. On the contrary, for excessive live load and larger panels, providing ACI Code minimum thickness may not be adequate. In such conditions, deflection calculations should be mandatory to decide a higher thickness. In this paper a simplified procedure of deflection calculation has been presented to help designer selecting slab thickness other than ACI specified value. The design charts have been found to model experimental results realistically. The use of the design charts in calculating short and long-term deflections are illustrated with an example.

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