

# Predicting downstream hydraulic geometry of the Gorai river

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## Abstract

The Gorai river is the main source of fresh water inflow into the southwest region of Bangladesh. The geometry of the Gorai river in terms of bank-full width, average flow depth and mean flow velocity have been examined. The fit of the observed and predicted depth and width is satisfactory. The fit of the observed and predicted velocity is poor. The goodness-of-fit of the depth improves significantly after applying correction factor.

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## 1. Introduction

The Gorai river is the main distributary of fresh water from the Ganges to the southwest region of Bangladesh. During the last decades, the low flow characteristics of the Gorai has changed significantly, The Gorai has been virtually dry during almost the entire dry season since 1989 due to substantial sedimentation of the river at its offtake from the Ganges. Morphological developments in the Ganges river basin, the operation of the Farakka Barrage on the Ganges further upstream on the Indian side of the border and other human interference on the rivers in the region have been sited as reasons for deterioration of the Gorai (FAP 24, 1996). FAP 4 (1993) study reviewed the development from 1973 to 1992 based on satellite images. The discharge distribution at an offtake is to a large extent determined by the hydraulic capacity along the entire offtake river branch and relative discharge can therefore be stage dependent. The sediment transport distribution, in addition, may be influenced by the local planform and sediment transport condition of the main river.

Islam (1996) studied the morphological characteristics of the Gorai-Madhumati river. He utilized time series data on water level and discharge. In addition sediment transport, rating curves, channel cross-section and channel alignment data were analysed. The relationships between meander parameters and channel width as well as the representative discharge was developed. EGIS (1999) found that during the last two and half decades the annual flow volume of the Gorai river is declining. Kader (2000) studied the effectiveness of pilot dredging in the Gorai river. Groot and Groen (2001) have found that the intervention by dredging should be included in any long term solutions for the restoration of the river and can be seen as an alternative for building any conventional hard construction in the river. Sarker et al. (2002) studied the morphological changes of the Gorai in response to the declining flow using remote sensing. They observed that in response to the changes in the hydraulic regime, morphological characteristics of the river have been changing as well. Garsdal et al. (2002) studied the use of mathematical models in connection with the Gorai river restoration project in Bangladesh. They found that the hydraulic performance and morphology of the Gorai river are strongly affected by the morphological conditions locally near the offtake, both in the Ganges and in the Gorai.

The Gorai is an alluvial river. Alluvial rivers are known to adjust their slope, width, depth and velocity to achieve stable conditions at a specified supply of water and sediment. Downstream hydraulic geometry relationships describe the shape of the bank-full alluvial channels in terms of bank full width, average flow depth, average flow velocity and channel slope. Considerable progress has been achieved in river mechanics after a century of field investigation on the geometry of alluvial rivers under equilibrium or in regime. Julien and Wargadalam (1995) give an extensive list of the contributors. Limitations of most existing methods relate to the simplified one-dimensional analysis of flow and sediment transport in alluvial channels. Two dimensional flows are not only important to define flow patterns in meandering and braiding channels, but also to determine the particle migration rate and the rate of alluvial channel deformation and thus the hydraulic geometry of alluvial channels. Julien and Wargadalam (1995) derived downstream hydraulic geometry equations by incorporating the concepts of secondary flows in curved channels and the three dimensional mobility of non-cohesive particles. In this study, the applicability of hydraulic geometry equations has been examined to predict the downstream hydraulic geometry of the Gorai river.

## 2. Methodology

Geometric data on width, depth, cross sectional area and hydraulic data on discharge, velocity of the Gorai river at Gorai Railway bridge have been collected from Processing and Forecasting Circle of Bangladesh Water Development Board (BWDB). The data covers from 1983 to 2002. The value of  $S$  has been taken as  $5 \times 10^{-5}$  (FAP 24, 1996). The data have been checked for possible inconsistency. The inconsistent data can be detected by plotting the stage against discharge of the annual hydrograph. Inconsistency in the discharge data has been detected by plotting stage and discharge data against cross-section and cross-section averaged velocity for every hydrological year. Karim (2004) checked the inconsistency of the data. Different geometric and hydraulic data have been analyzed to find the change over time.

Downstream hydraulic geometry relationships describing the shape of the bank-full alluvial channels in terms of bank full width, average flow depth, average flow velocity and channel slope have been examined by the formulation given by Julien and Wargadalam (1995) which overcomes the limitations of most existing methods which

relate to the simplified one-dimensional analysis of flow and sediment transport in alluvial channels. The predicted parameters have been compared with the observed one to find the applicability of the formulations in the Gorai river. The lack of the fit has been explained. The prediction of the geometric parameters has been made.

### 3. Formulation of geometric characteristics

Julien and Wargadalam (1995) analytically defined the equilibrium downstream hydraulic geometry relationships of deformable alluvial channels. The innovative aspects of this include the concepts of secondary flows in curved channels, and the three-dimensional mobility of non-cohesive particles. They theoretically derived the downstream hydraulic geometry equations.

#### 3.1 Flow characteristics in alluvial channels

The downstream hydraulic geometry – regime geometry – of non-cohesive alluvial channels can be determined from the stability of sediment particles under two dimensional flow conditions. The downstream hydraulic geometry is defined in terms of surface channel width  $W$ , average flow depth  $h$ , average flow velocity  $U$  and channel slope  $S$ . Under steady uniform bank-full flow condition, the dominant discharge  $Q$  is

$$Q = WhU \quad (1)$$

where, the mean velocity vector  $U$  is taken normal to the cross-sectional area. When considering turbulent flows over hydraulically rough boundaries of sediment size  $d_s$ , the resistance equation becomes

$$\frac{1}{\sqrt{f}} = b \left( \frac{h}{d_s} \right)^m \quad (2)$$

where, the value of exponent  $m$  is the following function of the relative submergence  $h/d_s$ :

$$m = \frac{1}{\ln \left( \frac{12.2h}{d_s} \right)} \quad (3)$$

In channels with coarse bed materials, the exponent  $m$  increases rapidly at low values of relative submergence ( $h/d_s < 10$ ). Thus, the average flow velocity follows from (3) as proposed by Einstein and Chien (1954)

$$U = b\sqrt{8g} \left( \frac{h}{d_s} \right)^m h^{1/2} S^{1/2} \quad (4)$$

where, the exponent  $m$  increases with decreasing relative submergence.

The downstream bed shear stress  $\tau_\theta$  applied in straight open channels under steady uniform flow conditions is a function of the bed slope  $S$ , the mass density  $\rho$  and the hydraulic radius  $R_h = Kh$

$$\tau_\theta = K\rho ghS \quad (5)$$

### 3.2 Two-dimensional flow in alluvial channels

Secondary circulation in curved channels is generated through a change in downstream channel orientation. The following formulation for the deviation angle  $\lambda$  is used, in which the values of transverse resistance exponent  $p$  and coefficient of transverse resistance equation  $b_r$  accommodate at wide spectrum of conditions pertaining to the secondary circulation in alluvial channel bends as proposed by Rozovskii (1961).

$$\tan \lambda = b_r \left( \frac{h}{d_s} \right)^p \frac{h}{R} \quad (6)$$

### 3.3 Particle stability analysis in alluvial channels

The stability of non-cohesive particles in straight alluvial channels is described by the relative magnitude of the downstream shear force and the weight of the particle. The ratio of these two forces defines the longitudinal mobility factor, also called the Shields number  $\tau_\theta^*$

$$\tau_\theta^* = \frac{\tau_\theta}{(\rho_s - \rho)gd_s} \quad (7)$$

where,  $\rho_s$  = mass density of sediment particles.

### 3.4 Downstream hydraulic geometry equations

The downstream hydraulic geometry relationships for non-cohesive alluvial channels under hydraulically rough turbulent flows are derived by combining the following four fundamental relationships: (1) flow rate [1]; (2) resistance to flow [4]; (3) particle mobility [7 and 5] and (4) secondary flow [6]. These four equations are combined and solved for channel width  $W$ , flow depth  $h$ , average flow velocity  $U$ , and slope  $S$ , and are written as a power function of discharge  $Q$ , sediment size  $d_s$ , Shields parameter  $\tau_\theta^*$ , and deviation angle  $\lambda$ . The equilibrium downstream hydraulic geometry of non-cohesive alluvial channels in terms of average flow depth  $h$  (in m), surface width  $W$  (in m), average flow velocity  $U$  (in m/s) and friction slope  $S$  is found from the following equations

$$h = 0.133Q^{1/(3m+2)} d_s^{(6m-1)/(6m+4)} \tau_\theta^{*-1/(6m+4)} \quad (8)$$

$$W = 0.512Q^{(2m+1)/(3m+2)} d_s^{(-4m-1)/(6m+4)} \tau_\theta^{*(-2m-1)/(6m+4)} \quad (9)$$

$$U = 14.7Q^{m/(3m+2)} d_s^{(2-2m)/(6m+4)} \tau_\theta^{*(2m+2)/(6m+4)} \quad (10)$$

$$S = 12.4Q^{-1/(3m+2)} d_s^{5/(6m+4)} \tau_\theta^{*(6m+5)/(6m+4)} \quad (11)$$

#### 4. Examination of the applicability of the formulation

The applicability of the geometric formulation has been examined with the Gorai river data. As mentioned earlier the hydraulic and geometric data of the Gorai river have been collected from BWDB. Other types of data e.g. sediment size and slopes have been taken from reports (FAP 24, 1996). Based on the data, the downstream hydraulic geometry equations have been used to predict width, depth and velocity. The value of the exponent  $m$  is found to be in the range of 0.072~0.089 for the Gorai river (Karim, 2004).

Taking all the years into consideration, the observed depth, width and velocity have been plotted against the computed ones. The plot of predicted depth against the observed depth is shown in Figure 1. The overall fit of depth is satisfactory especially for lower depths although the predicted depths are overestimated. A multiplying factor of 0.75 has been applied to the predicted depth and plotted against observed depth as shown in Figure 2. It can be seen that the fit has been improved significantly. The plot of predicted width against the observed width is shown in Figure 3. It is seen that the fit between observed and predicted width is satisfactory during lower widths (below 450 m) when the river flow is near to the bank-full flow. Above 450 m of width, the prediction overestimates to a large extent. The plot of predicted velocity against the observed velocity is shown in Figure 4. The fit between observed and predicted velocity is rather poor. The predicted velocity overwhelmingly underestimates the observed velocity. As magnitude of velocity is heavily dependent on slope, this might be a cause as the bed slope has been chosen as constant. This, however, needs further study.

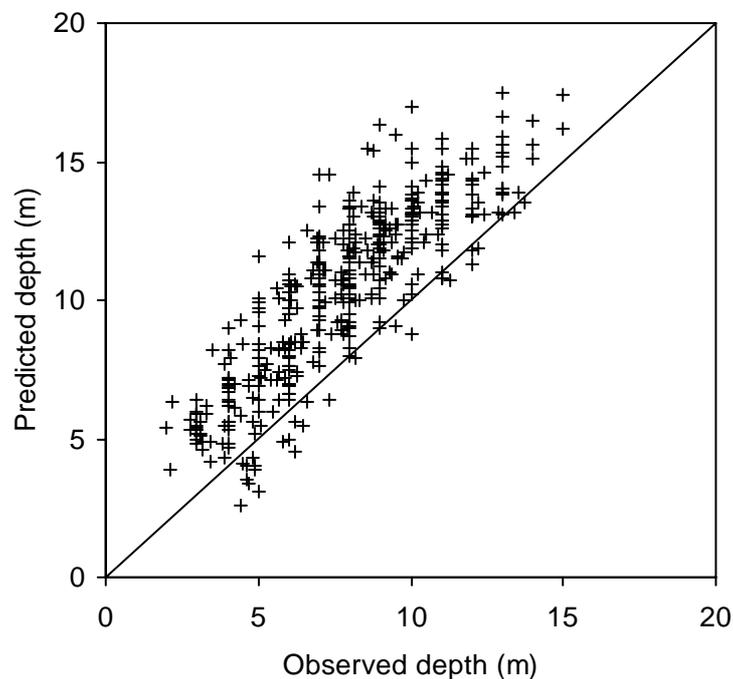


Fig. 1. Comparison of the observed and predicted flow depth

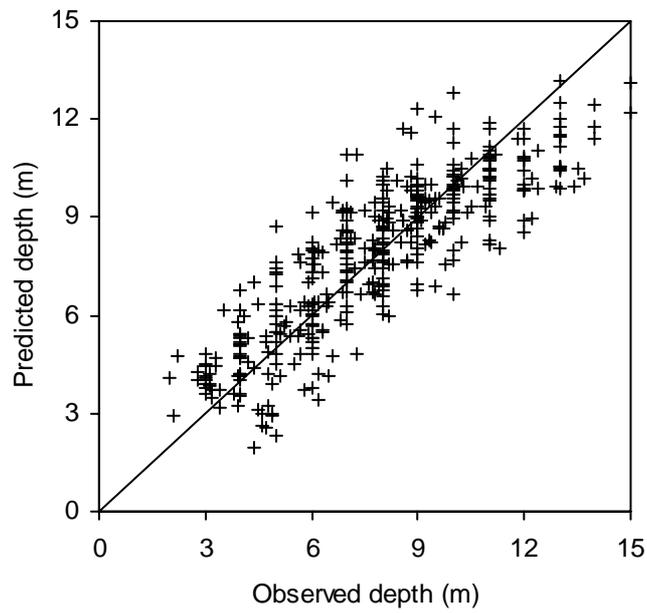


Fig. 2. Comparison of the observed and predicted flow depth after applying the correction factor

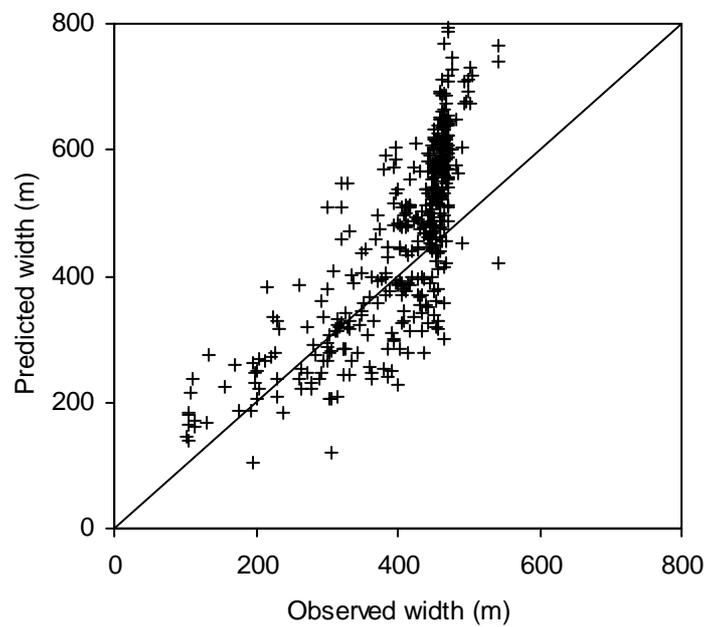


Fig. 3. Comparison of the observed and predicted width

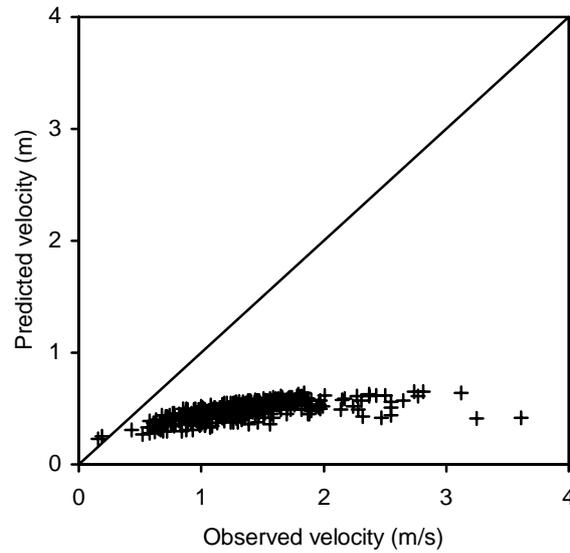


Fig. 4. Comparison of the observed and predicted velocity

### 5. Performance evaluation by the goodness-of-fit

The discrepancy ratio and standard deviation have been used to indicate the accuracy of the goodness-of-fit. The discrepancy ratio indicates the goodness-of-fit between the predicted and observed results. One of the ways to measure the goodness-of-fit is the use of average discrepancy ratio and standard deviation based on the average value of the logarithm ratio between computed and measured results using the following parameters:

$$D_i = \log\left(\frac{\psi_c}{\psi_m}\right) = \log \psi_c - \log \psi_m \tag{12}$$

$$\bar{D}_a = \frac{\sum_{i=1}^j D_i}{j} \tag{13}$$

$$\sigma_a = \sqrt{\frac{\sum_{i=1}^j (D_i - \bar{D}_a)^2}{j-1}} \tag{14}$$

where,  $D_i$  = discrepancy ratio based on logarithm ratio,  $\psi_c$  = computed value,  $\psi_m$  = measured value,  $\bar{D}_a$  = averaged discrepancy ratio based on logarithm ratio,  $j$  = total number of data used,  $\sigma_a$  = standard deviation based on the logarithm ratio,

For a perfect fit,  $\bar{D}_a = 0$  and  $\sigma_a = 0$ . Comparisons between predicted and observed values based on the average logarithm ratio are summarized in Table 1. It can be seen from that the goodness-of-fit of the depth and width is satisfactory while that of the velocity is poor. If a multiplying factor of 0.75 is applied to the predicted depth, then the fit is improved significantly as the average discrepancy ratio changes from 0.133 to 0.010. After applying the transformation to the predicted velocity, the fit is improved to a great extent.

Table 1  
Summary of comparisons between predicted and measured hydraulic geometry parameters

Parameter	$\bar{D}_a$	$\sigma_a$
Depth	0.133	0.093
Depth (correction factor)	0.010	0.093
Width	0.051	0.097
Velocity	0.423	0.110

## 6. Conclusion

The geometry of the Gorai river in terms of bank-full width, average flow depth and mean flow velocity have been examined. The downstream hydraulic geometric equations have been applied to the Gorai river data to examine the applicability of equations. The fit of the observed and predicted depth is satisfactory. The fit of observed and predicted width is good specially at lower depth. The fit of the observed and predicted velocity is poor. The fit of the depth has been improved significantly by correction factor.

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## Notations

- a = Coefficient of logarithmic resistance equation  
b = Coefficient of resistance equation

br	= Coefficient of transversal resistance equation
ds	= Sediment size
Di	= Discrepancy ratio based on logarithm ratio
$\overline{D}_a$	= Averaged discrepancy ratio based on logarithm ratio
f	= Darcy-Weisbach friction factor
g	= Gravitation acceleration
h	= Average flow depth
j	= Total number of data used
K	= Dimensionless submerged density of sediment
m	= Exponent of resistance equation
p	= Transverse resistance exponent
Q	= Dominant discharge
Rh	= Hydraulic radius
S	= Channel slope
U	= Average flow velocity
W	= Channel width
$\sigma_a$	= Standard deviation based on the logarithm ratio
$\rho$	= Mass density of fluid
$\rho_s$	= Mass density of sediments
$\lambda$	= Flow deviation angle
	= Kinematic viscosity of fluid
$\tau_\theta^*$	= Shields number
$\tau_\theta$	= Downstream bed shear stress
$\psi_c$	= Computed value
$\psi_m$	= Measured value

### Abbreviations

BWDB	= Bangladesh Water Development Board
FAP	= Flood Action Plan