

# A novel approach of modeling precast concrete frame joint: modeling, numerical simulation and performance evaluation

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## Abstract

A mathematical model capable of analyzing multistoried building frames incorporating semi-rigid beam-column connections has been developed. The model is linear type and iterative in nature developed for multistoried building frames with connections which are based on maximum end moment of frame. A mathematical relationship is developed and used for analysis and design of joints of a particular type. The linear moment-curvature ( $M - \theta$ ) relationship for this particular type of connection has been formulated by using elementary solid mechanics. Using this relationship investigations have been carried out to study the behavior of precast building frame such as connection flexibility and its effect on internal distribution of forces. A parametric study with the connection is also carried out. A comparison of overall behavior of frames with rigid and partially restrained joints is made. Finally a tentative recommendation regarding the use of the particular type of joint is made.

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*Keywords:* precast frame, moment-curvature relation, connection flexibility, internal distribution of forces.

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## 1. Introduction

### 1.1 General

Structural prefabrication is a new construction procedure which during the last few years has greatly enlarged the potentialities of reinforced concrete. It consists in building a complex structure by connecting prefabricated concrete elements, which collaborate to

give structural strength to the whole. The elements may be either of reinforced concrete or of Ferro-cement. It is easily understood that a combination of these two materials offers new and interesting possibilities, and that prestressing may increase their potential. Besides its technical advantages, prefabrication possesses interesting esthetic-architectural characteristics deriving from the inherent lightness of prefabricated structures and from the speed of construction typical of mass-produced elements. Moreover, prefabrication allows the use of elements of complicated shape without construction difficulties or expensive formwork, since each form is used to pour a large number of elements.

### 1.2 Problem identification

The connection of the constituent members of precast construction plays an important role in affecting the behavior of the complete structure because the amount of moment-transfer is controlled by the joint characteristics. In the analysis and design of precast building frames, it is customary to represent joint behavior by an idealized model, either as a rigid joint or as a pinned joint. A number of experimental investigations have established that these two extreme assumptions are, strictly speaking, unattainable in practice. In reality most connections are semi-rigid in nature and possess some amount of rotational stiffness. Although connections constitute a small percentage of the weight of the structure, they have relatively high labor content and hence represent a substantial percentage of the total framing cost; as such they constitute one of the most significant elements to be considered in the overall economies of construction of a tall building. Rigid connections are associated with heavy connection elements along with fully developed welds or large connectors involving high cost. On the other hand simple framing requires some other provisions like bracing systems for carrying lateral loads and thus involves additional cost. Although precast semi-rigid construction is recognized by the codes and is economical, it has not become a viable type of construction due to lack of confidence about its behavior. No specific design procedure for precast construction has been recommended yet. In order to overcome these difficulties, a realistic mathematical model, accounts for the connection flexibility effect in a prefabricated building frame is proposed here.

### 1.3 Behavior of connection

The prediction of joint rotational behavior is a preliminary step in the analysis of semi-rigid frames. Numerous investigations into the behavior of beam-to-column connections have been reported during past few decades. The flexural behavior of a connection is best represented by the relationship between  $M$ , the moment transmitted through the connection, and  $\theta$ , the relative rotation of the two members fastened by the connection. To realize a rational design method for flexibly jointed frames, availability of moment-rotation relations or  $M - \theta$  curves of practical connections is a prerequisite. Since it is not convenient to use the experimental results in the analysis or design of a frame, it is important to model the connection  $M - \theta$  behavior mathematically so that a reasonable estimate of the rotational stiffness of the connection at any level of moment can be made. The rotational deformation ( $\theta$ ) is customarily expressed as a function of the connection moment ( $M$ ). The rotation ( $\theta$ ) represents the change in angle between the beam and the column from its original configuration.

#### 1.4 Methods of analysis of semi-rigid frames

Attempts to include semi-rigid joint action in the analysis include a wide range of work; from modification of traditional methods of analysis of rigid frames to the formulation of classical finite element models. The slope deflection equations were used to represent members with semi-rigid joints at ends by modifying the coefficients of the usual rigid case (Johnston and Mount, 1942). The procedure is otherwise the same as the conventional slope deflection method. The modified moment distribution method also follows the same procedure as the conventional one, with the only difference being that different distribution and carry over factors are used with semi-fixed end moments. The other basic method like the Method of Three Moments has also been modified to take account of the semi-rigid nature of the beam-column connections. In all of these modifications linear connection stiffness was assumed, which is taken as the initial slope of the connection  $(M - \theta)$  curve. This simplification, however, remains a formidable shortcoming because of the fact that, in most cases, non-linearity in connection behavior starts even at a small load application. These methods have not become popular because of this limitation coupled with the complexities in their use. The application of computers has made it possible to represent the joint behavior in a more refined and accurate manner. Most approaches to the analysis of semi-rigid frames have been developed by introducing one or more discrete spring elements (Ackroyd, 1979; Goverdhan, 1984; Cosenza et al, 1984; Lui, 1985; Shukla, 1986) to simulate the joint response. Each of these springs can be assigned a predetermined force-displacement relation representing the axial, shear and flexural behavior of the joint. Any type of constitutive law can, in principle, be assumed: linear elastic, non-linear elastic or inelastic, or by directly modifying the member stiffness relationships to account for the partial rotational restraining effect of the connections (Nethercot, 1974; Allen and Bulson, 1980; Wang, 1983; Lee, 1987).

The former approach has the disadvantage that the total number of degrees of freedom required to model the deformed configuration of the structure increases significantly. Despite wide-spread availability of microcomputers, these approaches are more suitable for an academic setting than day-to-day design office practice the main features of some of the important developments in the recent years are discussed here Cosenza, De Luca and Faella (1984) also developed a computer program which utilizes the stiffness method of analysis and includes second order effects. Semi-rigid joints were modeled as extra elements consisting of short rigid segments and springs with axial, shear and rotational stiffness. Anderson and Lok (1985) developed a method of analysis to incorporate the influence of connection flexibility into the analysis of plane frames. Second order effects considered in this elastic analysis procedure. In the analysis the rotations at any connections except real pins are initially assumed to be zero. Using conventional rigid frame analysis, the displacement and rotations are calculated and hence the member end reactions are obtained using slope deflection equations. Connection  $(M - \theta)$  characteristics are then used to assess connection rotations and these are used to amend the applied load vector. Using this new vector of applied loads, a new vector of displacements and thus new member end reactions are obtained. The procedure is repeated until the convergence is achieved. Chen and Lui (1985) employed the stiffness method in which the element matrices were derived on the basis that an element with two semi-rigid joints at its ends is treated as a sub-structure. The sub-structure consists of three sub-elements: two joint elements and one beam-column element. Stability functions derived by Lui (1985) were used to account for the presence of axial forces in the beam-column elements and an incremental iterative type of analysis was used. The

$(M - \theta)$  data for the connections were represented by an exponential function. Poggi and Zandonini, (1985) reported the development of a program by modifying the program developed by Corradi and Poggi (1985) to include the effect of semi-rigid joints. In this program  $(M - \theta)$  data for the connection was modeled by a series of straight lines. It is based on small deflection theory - which obviously affects its performance for the analysis of flexibly connected sway frames, where the occurrence of large displacements is commonly encountered. This analysis program includes neither material nor geometrical imperfections and is capable of handling column bending about the major axis only. Lee, (1987) developed a analytical approach which is based on the slope-deflection method in which the equilibrium equations are written with respect to the deformed shape of the structure (Galambos, 1968) and involves the use of stability functions to reflect the effect of member axial forces exactly. Jones, (1980) developed a computer program to trace the load deflection behavior of an isolated column with semi-rigid joints up to its failure load. This finite element program includes both geometric and material non-linearity. The column was assumed to be connected to infinitely rigid beams through semi-rigid joints (i.e. beam stiffness were not included). A non-linear  $(M - \theta)$  relationship was utilized. Jones was the first to use the B-spline technique to model the connection  $(M - \theta)$  relationship. He concluded that use of even the most flexible connections may improve the buckling load of the column considerably. Following Jones (1980) work, Rifai (1987) developed a program to analyze a beam-column sub assemblage. This finite element formulation again considers both geometric and material non-linearity; the influence of residual stresses and geometric imperfections is included. He concluded that the stiffness of the beam and the performance of the joint stiffness both influence the restraint for the column. The accuracy of the program was verified by experimental work undertaken by Davison (1987).

### 1.5 Objectives and methodology

The present paper aims at developing an appropriate mathematical model for the analysis of precast multistoried building frames incorporating the connection flexibility effects and to perform static and dynamic analysis on the building frames changing various governing parameters of connection influencing the behavior of such frames. Also to investigate the distribution of internal forces in the members and the load-deflection behavior of the frames for different types of connection and loading conditions. To this end a comparison has been made of the proposed model with the rigid connections. The present study has examined the behavior of medium rise multi-storied semi rigid precast frames. For this purpose two dimensional plane frames, typical 10 storied, have been studied. The frames have been subjected to static loading - wind load and gravity load. And dynamic loading in the form of response spectrum has been considered in this study. Frames have been assumed to consist of ideal members having no residual stress or initial imperfection. The global imperfection of frames has also been ignored. Inclination of members has not been considered in this study. Only major axis bending of members has been considered. In this study only linear elastic analysis is performed.

## 2. Mathematical modeling

### 2.1 Connection details

The present is intended with attention mainly to the beam-column connection of precast building structures, where the beam is precast, and the column is cast-in-situ. The beam is connected with a plate at the end which is also welded to the longitudinal

reinforcement of the beam. The steel plate in column is welded to the dowel bars placed transversely in column as shown in Figure 1 and Figure 2. These connections are assembled by bolting the steel plate at the top and bottom portion of the beam to the plate of the column. Washer has to be incorporated in the joint to avoid point contact and cracking of the concrete. It must be ensured that the detailing of connection provides full transfer of forces across the beam- column interface.

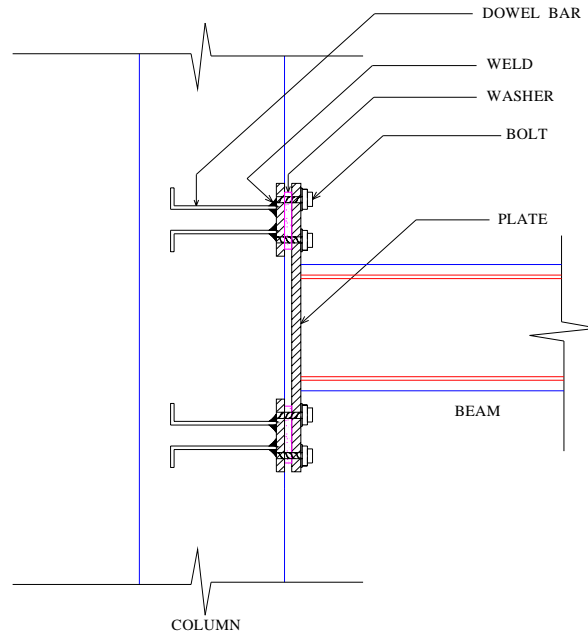


Fig. 1. Connection details without stiffener

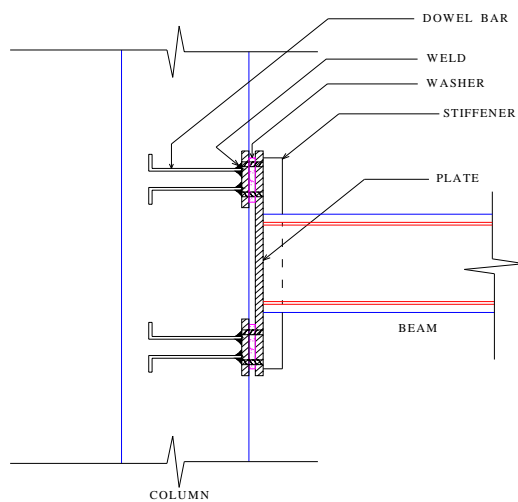


Fig. 2. Connection details with stiffener

## 2.2 Moment-rotation relation

The flexural connection behavior is represented by the moment rotation relationship, which relates the moment transmitted by the connection of the relative rotation of the connecting members. The moment-rotation curves for all type of connection are of non linear type. In our study, we assume that moment-rotation curve is linear type. The connection size parameters were used to generate the force-deformation characteristics for various connections. There are two ways that connection moment-rotation relationships can be incorporated into structural analysis program. The moment-rotation information of every connection of every type can be stored. Since for any given type of connection, there are a number of "size parameters" such as beam size, plate and bolt thickness, stiffener size, etc this requires the storing of an extremely large amount of information. On the other hand as the moment-rotation characteristics for all connections of a given type are quite similar, a standardized moment-rotation relationship can be derived as function for the size parameters for that type of connection. The moment-rotation characteristics for a particular type of connection can then be generated by substituting its size parameter into the relationship. The later procedure is used in our analysis. The procedure involves the representation of the moment-rotation relation for all connection of a given type, in the following form:

Type1: Rotational deformation of connection due to elongation of bolt as shown in Fig. 3

Type2: Rotational deformation of connection due to bending of plate as shown in Fig. 4

Now from the elementary solid mechanics the rotation due to elongation of bolt can be obtained as follows:

$$\Delta = \frac{M}{h} \frac{(t + t_w)}{N_b A_b E_s} \quad (1)$$

$$A_b = \frac{M/h}{N_b f_{b,allow}} = \frac{\pi}{4} d_b^2 \quad (2)$$

$$d_b = \sqrt{\frac{4M}{\pi h N_b f_{b,allow}}} \quad (3)$$

$$\theta_1 = \frac{\Delta}{h} = \frac{4M(t + t_w)}{\pi N_b h^2 d_b^2 E_s} \quad (4)$$

where,

$t$  = Thickness of plate

$t_w$  = Thickness of the washer

$N_b$  = Total number of bolt

$E_s$  = Modulus of elasticity of steel

$d_b$  = Dia of bolt

$\Delta$  = Deflection of the plate

$M$  = Applied moment (kN-m)

$h$  = Moment arm of the resulting bolt

$A_b$  = Area of the bolt

$f_b$  = Allowable stress of bolt

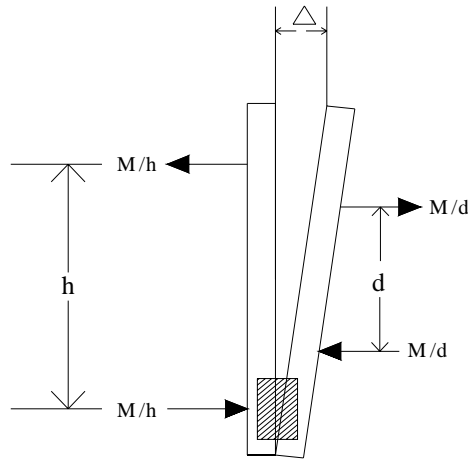


Fig. 3. Elongation of the bolt

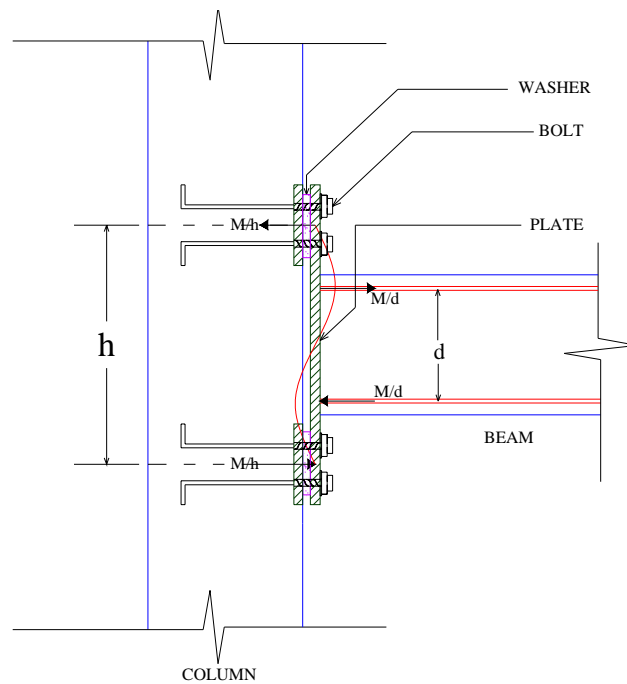


Fig. 4. Rotation due to bending of the plate

In an idealized case, the moment due to bending of the plate produces tension on the upper portion and compression on the lower portion of neutral axis. The tension developed by the top reinforcement is resisted by the bolt and the compression developed by the bottom reinforcement and the concrete of beam is resisted by the bolt and the compressive strength of the column concrete. For simplicity of the analysis, the concrete contribution is neglected. In order to determine the rotational deformation of

plate without stiffener due to bending, the analytical approach is based on the moment-area method as shown in Fig. 5.

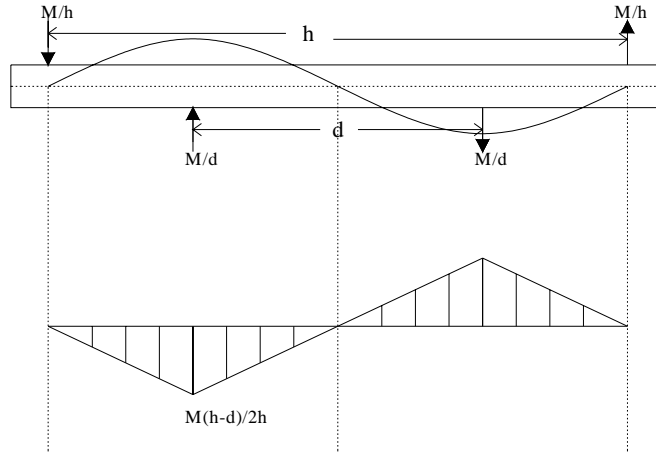


Fig. 5. Rotational deformation due to bending of the plate

$$E_s I_{plate} \theta_2 = M \left[ \frac{(h-d)^3}{12h^2} + \frac{d(h-d)(3h-2d)}{24h^2} \right] \quad (5)$$

$$\theta_2 = M \left[ \frac{2(h-d)^3 + d(h-d)(3h-2d)}{24h^2 E_s I_{plate}} \right] \quad (6)$$

So the total rotation is obtained by adding the Eqns 4 and 6 as,

$$\theta = \theta_1 + \theta_2 = M \left[ \frac{4(t+t_w)}{\pi N_b h^2 d_b^2 E_s} + \frac{2(h-d)^3 + d(h-d)(3h-2d)}{24h^2 E_s I_{plate}} \right] \quad (7)$$

This is the  $M - \theta$  relation of precast beam column joint. This relation makes the analysis to properly model and represent connection behavior,

where,

$b$  = Width of the plate

$d$  = Effective depth of the beam

$\theta_2$  = Rotation due to bending of the plate

$\theta$  = Total rotation

$I_{plate} = \frac{bt^3}{12}$  = Moment of inertia of the plate



The slope of  $M - \theta$  curve gives the stiffness of the joint. For plate with stiffener  $I_{plate}$  should be replaced by  $I_{sp}$  the moment of inertia of stiffened plate as,

$$I_{sp} = (N_{br} + 1) \left[ \frac{t_s h_s^3}{12} + t_s h_s \left( \frac{h_s}{2} - \bar{y} \right)^2 \right] + \frac{bt^3}{12} + bt \left( \frac{t}{2} + h_s - \bar{y} \right)^2 \quad (8)$$

where

$$\bar{y} = \frac{(N_{br} + 1)t_s h_s \frac{h_s}{2} + bt \left( h_s + \frac{t}{2} \right)}{(N_{br} + 1)t_s h_s + bt} \quad (9)$$

$I_{sp}$  = Moment of inertia of stiffened plate

$b$  = Width of column (width of plate)

$t_s$  = Thickness of the stiffener

$h_s$  = Height of the stiffener

$N_{br}$  = Number of bolt in one row

$\sigma_{allow}$  = Stress in plate

In order to find the thickness of plate to resist applied moment, an iterative procedure is used which is as follows:

If

$$\bar{y} < (t + h_s - \bar{y}) \quad (10)$$

$$I_{sp} = \frac{M(h-d)\bar{y}}{2h\sigma_{allow}} \quad (11)$$

$$\frac{bt^3}{12} = \left\{ \frac{M(h-d)\bar{y}}{2h\sigma_{allow}} - (N_{br} + 1) \left[ \frac{t_s h_s^3}{12} + t_s h_s \left( \frac{h_s}{2} - \bar{y} \right)^2 \right] - bt \left( \frac{t}{2} + h_s - \bar{y} \right)^2 \right\} \quad (12)$$

Else

$$\bar{y} < (t + h_s - \bar{y}) \quad (13)$$

$$\frac{bt^3}{12} = \left\{ \frac{M(h-d)(t+h_s-\bar{y})}{2h\sigma_{allow}} - (N_{br} + 1) \left[ \frac{t_s h_s^3}{12} + t_s h_s \left( \frac{h_s}{2} - (t+h_s-\bar{y}) \right)^2 \right] \right\} \quad (14)$$

$$\left. \begin{matrix} \\ -bt \left( \frac{t}{2} + h_s - (t+h_s-\bar{y}) \right)^2 \end{matrix} \right\}$$

### 2.3 Numerical simulation

In this study a typical 10 storey 3 bay 2-D frames with span length of 6 m and storey height 3 m was analyzed to show the applicability of the mathematical model and also to arrive at some important conclusion regarding the effect of semi-rigid connection on multi-storied building frame. For performing the analysis finite element method was used. Computer software ETABS version 8.02 was used for this analysis. The material properties used in this analysis are given in Table 1. The cross sectional properties of different elements of the frames are given in Table 2. The different types of loads and their combinations (BNBC, 1993) are given in Table 3. From these combinations the design loads are selected on the basis of maximum response. The joints are incorporated into the model following the concept of Chen and Lui (1985) in which the element are treated as sub-structure consists of three sub-elements: two joint elements and one beam-column element. The stiffness of the joint element is calculated from the height of the beam which is obtained from Eqn 7 as:

$$h = \sqrt[3]{\frac{12L}{4Eb} \left[ \frac{1}{\frac{4(t+t_w)}{\pi N_b h^2 d_b^2 E_s} + \frac{2(h-d)^3 + d(h-d)(3h-2d)}{24h^2 E_s I_{plate}}} \right]} \quad (15)$$

First the influencing parameters of the connection are identified and their effects on the behavior of the frames are investigated. A wide variety of connections, flexible to fairly rigid and perfectly rigid (fully restrained) are made from this parametric study. The details of the joints are presented in Table 4. Using these types of connection several investigations are performed to observed the behavior of the frames under service loads.

Table 1. Material properties for major components

Properties	Steel	Concrete
Modulus of elasticity (MPa)	206842	20684
Poison's ratio	0.3	0.15
Allowable stress (MPa)	248 (bolt) 331 (plate)	27.58

Table 2. Cross sectional properties of different elements

Section	Dimension			
Beam	300mm*450mm			
Column	500mm*500mm			
Cross sectional properties of towers				
Cross sectional properties	Area(mm <sup>2</sup> )	Moment of inertia(mm <sup>4</sup> )		Torsion constant(mm <sup>4</sup> )
		$I_{xx}$	$I_{yy}$	
Beam	135000	91125000	40500000	95080000
Column	250000	208328125	208328125	352078125

Table 3. Different types of load and their combinations

Gravity loading									
Dead load (kN/m <sup>2</sup> )							5.87		
Live Load (kN/m <sup>2</sup> )							2.00		
Wind load (kN)									
1 <sup>st</sup> storey	2 <sup>nd</sup> storey	3 <sup>rd</sup> storey	4 <sup>th</sup> storey	5 <sup>th</sup> storey	6 <sup>th</sup> storey	7 <sup>th</sup> storey	8 <sup>th</sup> storey	9 <sup>th</sup> storey	10 <sup>th</sup> storey
23.26	26.36	31.60	36.02	39.74	43.00	46.25	49.04	51.60	27.07
Combination 1					Dead load + Live load				
Combination 2					Dead load + Live load + Wind load				
Combination 3					Dead load + Live load + Earthquake load (applied as response spectrum UBC97 function)				

Table 4. Different types of joint

Joint type	No of bolts	Plate thickness (mm)	No of stiffener	Stiffener height (mm)
A	8	25.0	-	-
B	12	38.0	-	-
C	8	25.0	5	50.0
D	8	38.0	5	75.0
E	8	25.0	5	100.0

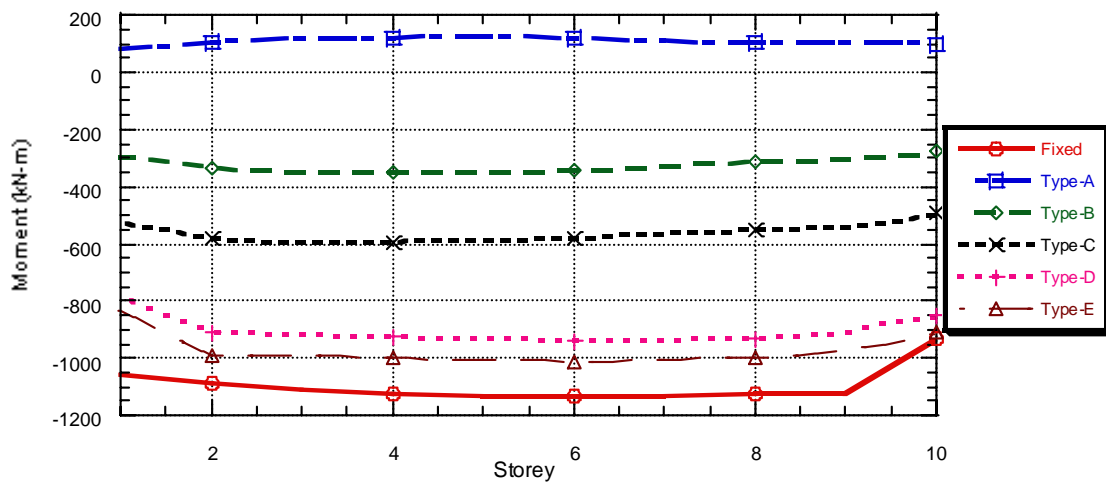


Fig. 6. Storey vs. joint moment for different types of joint

#### 2.4 Performance evaluation

Various types of joint are modeled ranging from very low stiffness to nearly rigid. From Figure 6 it is obvious that joint type A has very low stiffness as it produces positive moment at the joint which is a characteristic of simply supported beam having very low or no rotational stiffness at the end. Joint type E produces moment nearly equal to the fixed end moment. The rest of the joint type's produces moment in between simply supported and fixed which shows the semi-rigid characteristics of the proposed model. Figures 7 and 8 represents the beam positive moment for different types of joint, here it

also obvious that with the increasing of the stiffness of the joint the beam positive moment decreases. Figure 9 represents the relative rotation at various stories for different types of joint. Here it also shows that with the increasing of the stiffness of the joint the joint rotation decreases. Figure 10 shows sway produced under service loads for different types of joint, with the increasing of the stiffness of the joint the sway decreases. The types of joint have no effects on the vertical deflection as was evident from the Figure 11.

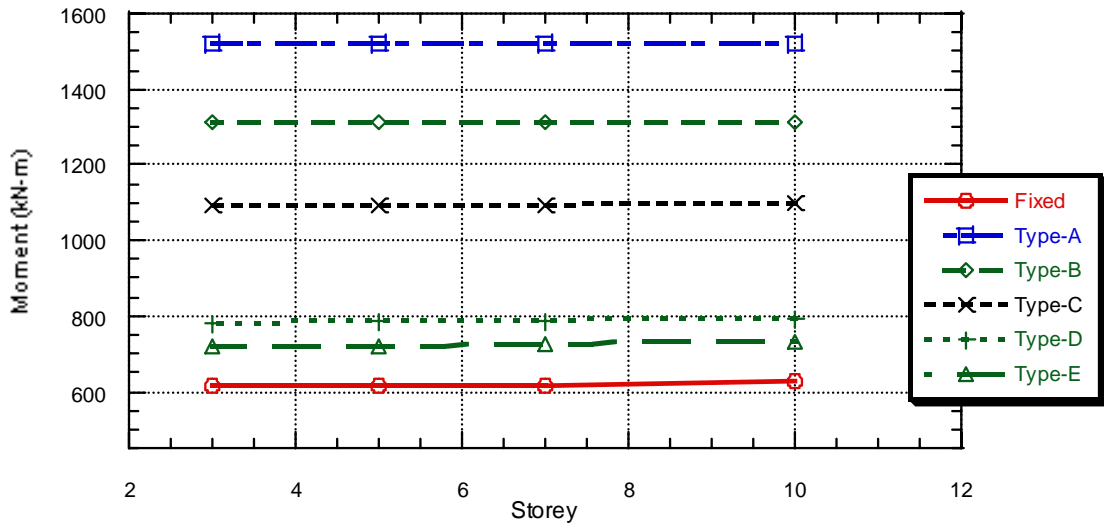


Fig. 7. Positive moment of beam at interior bay for different types of joint

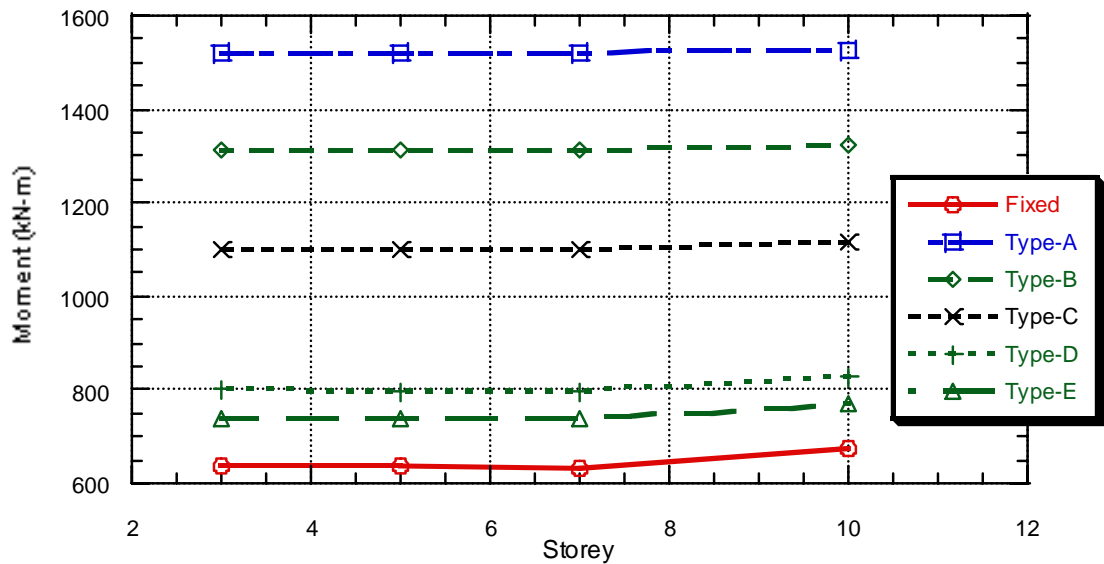


Fig. 8. Positive moment of beam at exterior bay for different types of joint

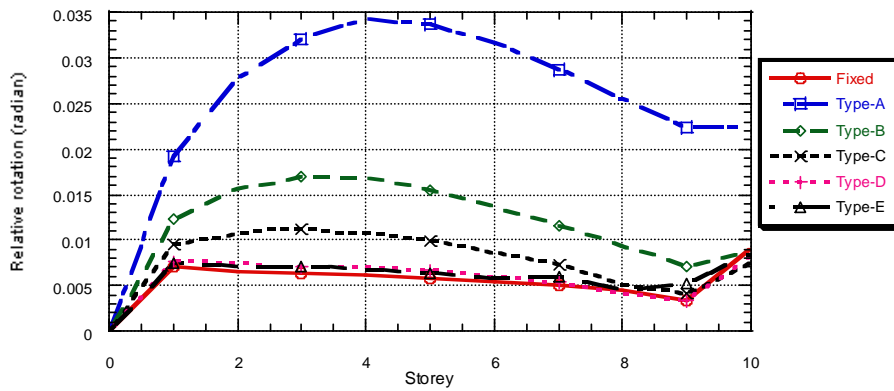


Fig. 9. Relative rotation (drift) vs. storey for different types of joint

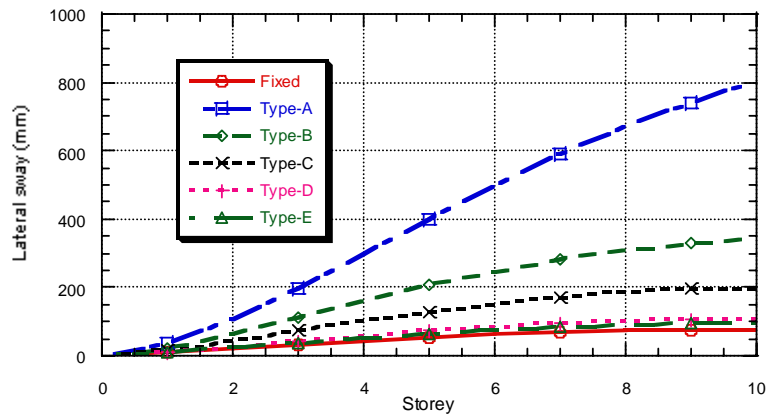


Fig. 10. Lateral sway vs. storey for different types of joint

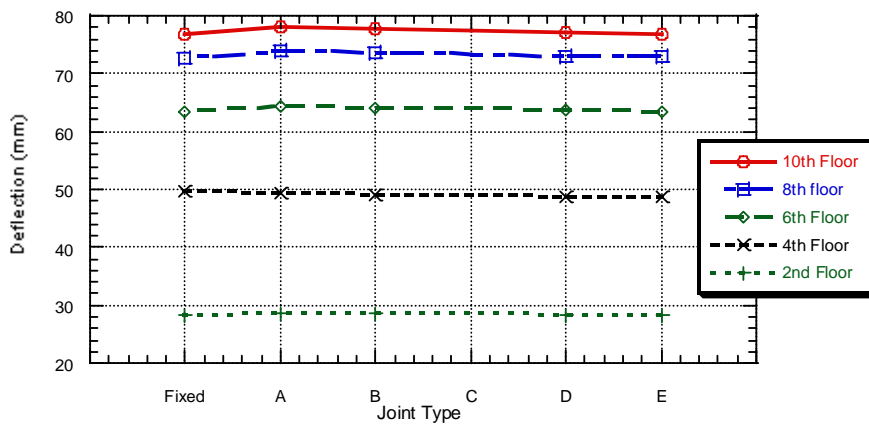


Fig. 11. Vertical deflection at different floors for different types of joint

### 3. Conclusion

The present study has yielded some tentative conclusions regarding the selection of connections based on behavior on the 2-D frame. For high rise building it would not be possible to use precast semi-rigid connection without providing any sway resisting system. Incorporation of the stiffener in a connection has a significant impact on moment carrying capacity and controlling sway. By increasing stiffener number and size, the moment and sway can be contained within acceptable limits.

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