

# Experimental study for sequent depth ratio of hydraulic jump in horizontal expanding channel

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## Abstract

The hydraulic jump within a lateral expansion is found to be of interest of the Hydraulic Engineers. These types of jumps may be expected at downstream of hydraulic structures where the incoming supercritical flow occupies a smaller width and leading to a wider tailwater. This paper describes an experimental investigation of a prediction model for computing the sequent depth ratio of hydraulic jump in abruptly expanding rectangular channels. Experiments have been conducted in the laboratory flume for the combinations of different expanding channels with different gate openings. The results of the experimental study were used to evaluate the prediction equation whose format is similar to the well-known Belanger equation for classical jump. The prediction equation is suggested to incorporate the effect of the abrupt expansion. This theoretically based equation is easy and simple to apply in design of enlarged stilling basin.

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*Keywords:* Prediction model, sequent depth ratio, froude number, expanding channel, laboratory flume.

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## 1. Introduction

The most important application of the hydraulic jump is in the dissipation of energy below sluices, weirs, gates, etc. so that objectionable scour in the downstream channel is prevented. The classical jump is formed in the regular rectangular channel in horizontal floor. In practice, the classical jump is being used for the design of regular type of stilling basin. But when the tailwater depth is inadequate to give a classical jump in a channel of constant width even with the aid of appurtenances and if it is not possible to depress the basin floor because of difficulties in excavation, then a lateral expansion remains the only possibility for guaranteeing the required dissipation of energy (Herbrand 1973). In such basins, there are mainly two problems faced by the field

engineers who monitor the performance of the design. One is the determination of sequent depth and the other is the estimation of energy loss (Agarwal 2001).

Hydraulic jumps in expanding channels have received considerable attention, although only limited informations on successful energy dissipation are available (Nettleton and McCorquodale 1989). Notable efforts have been made by Rajaratnum and Subramanya (1968), Herbrand (1973), Hager (1985), and Bremen and Hager (1993, 1994). After making several investigations, Herbrand (1973), and, Bremen and Hager (1994) separately developed equations for sequent depth ratio of abrupt expansion.

Matin et al. (1997) developed a Belanger's format prediction model for computing sequent depth ratio of hydraulic jumps in abruptly expanding channel based on the application of the one-dimensional momentum and continuity equations. Evaluation of the coefficients and exponents of the developed equation need experimental data for its use in practice. The validity range of the parameters used in the developed equation has also known from the experimental study.

This paper describes an experimental study (Hasan 2001) to evaluate a relationship for predicting sequent depth ratio of the prediction model. Equations presented will be helpful in calculating the sequent depth of the aforesaid jump. The experimentation of the sequent depths of hydraulic jump in an abruptly expanding, rectangular and horizontal channel is considered. The toe of the hydraulic jump is located just at the expansion section.

## 2. Theoretical formulation

The definition sketch for a hydraulic jump in an abrupt expansion is shown in Figure 1. In formulating the momentum equation, the pressure distribution is assumed as hydrostatic, the velocity distribution is considered as uniform, effect of turbulence and air entrainment are negligible, effect of frictional resistance from side walls is disregarded and the tailwater depth is assumed as the temporal mean value of its fluctuations. The momentum equation for the control volume between section 1 and section 2 (Figure 1) can be written as

$$P_1 + P_e - P_2 = \frac{\gamma}{g} Q(U_2 - U_1) \quad (1)$$

Defining,  $P_1 = 0.5\gamma b_1 h_1^2$ ,  $P_2 = 0.5\gamma b_2 h_2^2$  and  $P_e = 0.5\gamma(b_2 - b_1)h_1^2$ ; using the continuity equation  $Q = U_1 b_1 h_1 = U_2 b_2 h_2$  and introducing the dimensionless parameters  $D = h_2/h_1$ ,  $B = b_1/b_2$ , and  $e = h_e/h_1$  the equation (1) becomes

$$D^3 - D(1 + 2BF_1^2) + 2B^2F_1^2 = 0 \quad (2)$$

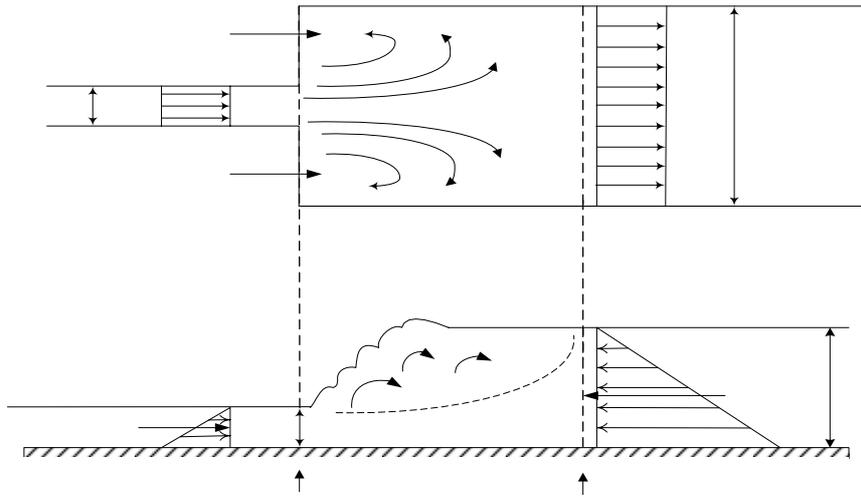


Fig. 1. Definition sketch of jump formed at expansion section.

The form of Equation 2 is also presented by Herbrand (1973). Using proper mathematical manipulation and simplifying, the Equation (2) will become in the following form (Matin et al., 1997)

$$D^2 + D - 2E^2 = 0 \tag{3}$$

where  $E$  is a modified Froude number which incorporates the effect of expansion and position of jump. This is expressed as

$$E^2 = k^{-1} F_1^2 \tag{4}$$

in which  $k$  is an assumed parameter to account for the effect of abrupt expansion of the channel on the jump depth. This parameter can be defined as

$$k = \frac{1 - D^{-1}}{B(1 - BD^{-1})} \tag{5}$$

Here,  $B$  is the expansion ratio ( $= b_1/b_2$ ), and  $h_1$  and  $h_2$  are the flow depth at the upstream and downstream of the channel respectively.

Matin et al (1997) developed also an equation based on experimental works to express  $k$  for the jump whose toe located at the expansion section. This equation is as follows:

$$k = 1 + 4.243 \left[ \log_{10} \left( \frac{1}{B} \right) \log_{10} (F_1) \right] \tag{6}$$

The sequent depth ratio  $D$  is then obtained by solving equation (3) as

$$D = \frac{1}{2} \left( \sqrt{1 + 8E^2} - 1 \right) \tag{7}$$

when  $B = 1$  which yields a value of  $k = 1$  and  $E = F_1$ , equation (7) reduces the well known Belanger equation for classical jump. The procedure is that the expansion ratio  $B$  for given Froude number  $F_1$  the Equation (7) can be used to calculate sequent depth ratio  $D$  using both Equation (5) and Equation (6).



Fig. 2. Front view of the experimental setup

### 3. Experimental investigation

Experiments have been conducted at the Hydraulics and River Engineering Laboratory of the Department of Water Resources Engineering, Bangladesh University of Engineering and Technology, Dhaka. All the investigations have been carried out in the laboratory flume having a 12.2 m overall length, a 0.3048 m depth and 0.3048 m width. The flume consists of an adjustable sluice gate, adjustable tailwater gate, water circulating system and water metering devices. For maintaining the exact expansion ratio, several constriction elements were installed in the stilling chamber in the flume. Just downstream of the sluice gate, two constriction elements were inserted along the direction of flow to make a reduced channel in the middle of the chamber (Figure 2). In the study, three different expansion ratios viz. 0.50, 0.67 and 0.83 were chosen to keep the constant downstream width  $b_2 = 0.3048$  m. There was no lateral movement of water between the constriction elements and the sidewalls. The length of the constriction element was based on the range of the stabilized classical jump formed on that flume. The tailwater depth was controlled by a vertical gate located at the downstream end of

the flume. Water issuing through an opening of the sluice gate, located some 4.0 m downstream from the water tank, formed the supercritical stream. During the experiments, the location of the hydraulic jump was controlled by the downstream gate and discharge. After the formation of stabilized jump, the readings of the sequent depths were taken carefully. Figure 3 shows a typical hydraulic jump in the expanding section (at  $B = 0.5$ ). A total of 44 test runs have been conducted of various flow conditions and measurements are shown in Table 1.

Table 1  
Summary of experimental flow conditions

Expansion Ratio	Sluice Gate Opening (cm)	Flow Rate ( $\text{m}^3/\text{sec}$ )	Inflow Froude Number	Upstream Depth (m)	Downstream Depth (m)	Run Numbers
0.50	3.6 – 7.3	0.0050-0.0125	1.33-2.65	0.032-0.058	0.066-0.134	1-15
0.67	3.6 – 7.3	0.0067-0.0167	1.70-3.48	0.026-0.052	0.063-0.127	16-30
0.83	3.6 – 7.3	0.0075-0.0196	2.05-3.06	0.026-0.046	0.076-0.158	31-44

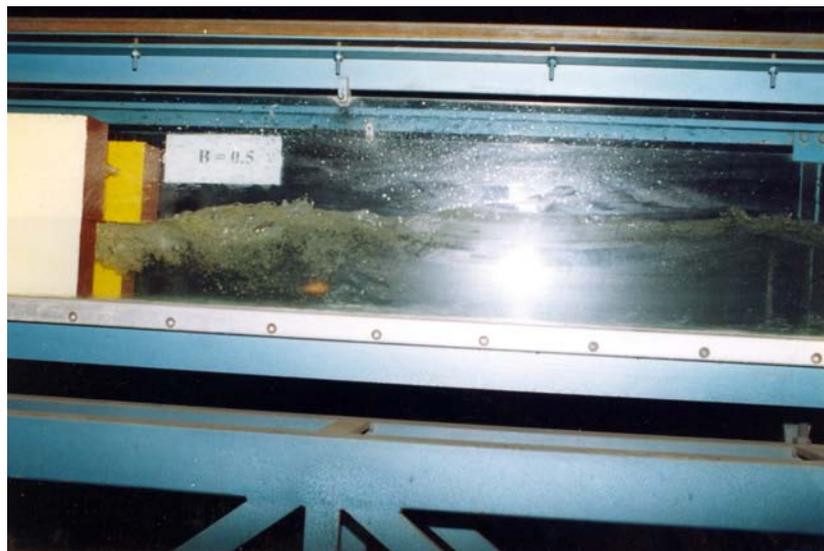


Fig. 3. Hydraulic jump in expanding section.

#### 4. Results and discussions

For each gate opening, different discharges were used to maintain the upstream depth below the critical depth of the channel flow. Figure 4 shows a relationship between the approach Froude number and the discharge for two different gate openings. It is seen that the inflow Froude number increases with the increases of discharges. Again the Froude number increases with the reduction of the sluice gate opening.

The values of parameter  $k$  from the laboratory experiment have been measured. The variation of parameter  $k$  with approaching Froude number for different expansion ratios is shown Figure 5. It is observed that the parameter value decreases with increasing expansion ratio.

For  $B = 0.50$ , the increasing trend of the curve is prominent and is almost linear when  $B = 0.833$ . The parameter  $k$  is more or less independent on the Froude number when expansion ratio  $B$  is more than  $0.667$ .

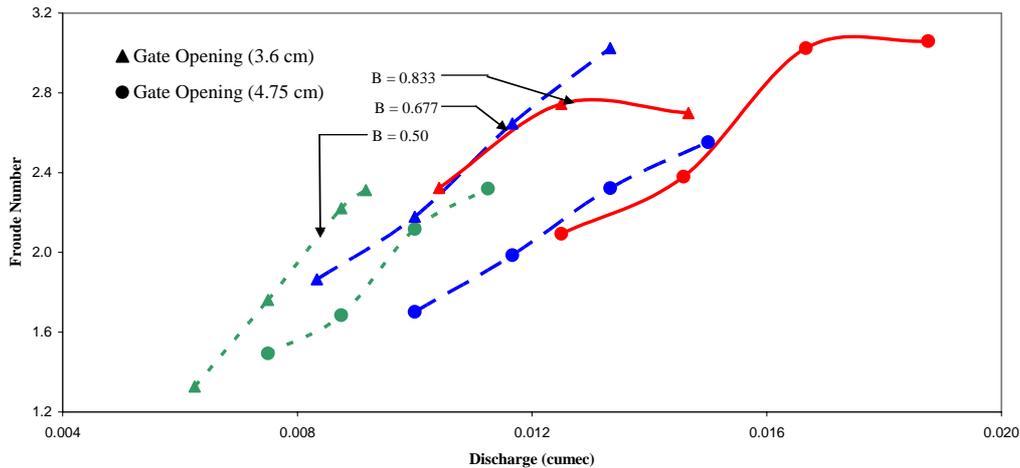


Figure 4: Discharge vs inflow Froude number for different gate openings

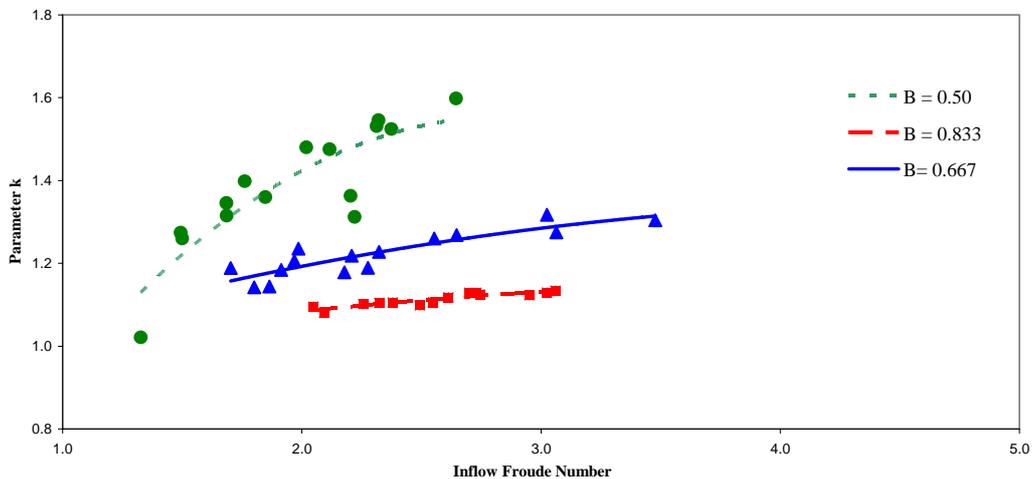


Figure 5: Variation of the parameter  $k$  for different expansion ratios

Figure 6 presents a comparison between Equation (5) and Equation (6) using the experimental data. To get a good agreement with the experimental data the prediction equation has been modified by using the coefficient 4.463 for  $B = 0.50$  and 3.493 for the higher values of  $B$  instead of 4.243 in the Equation (6). This adjustment produces fairly good agreement with the experimental data as shown in Figure 7. The predicted depth ratio after the modification of  $k$ , however, needs a multiplying factor varying from 1.03 to 1.17 for different gate openings. The predicted values after this adjustment

compliance with the observed values are shown in Figure 8. The variations of depth ratios with inflow Froude numbers are presented in Figure 9. It is observed that the depth ratio increases as Froude number increases. The line of classical jump is also shown ( $k = 1$ ). Physically, the classical jump requires more tailwater depth than other free jumps.

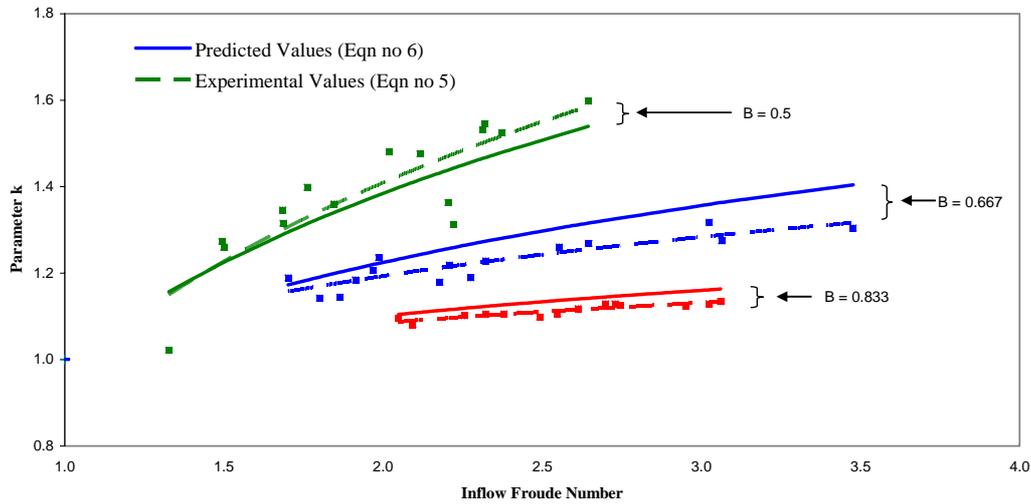


Figure 6: Comparison between measured and predicted parameter values

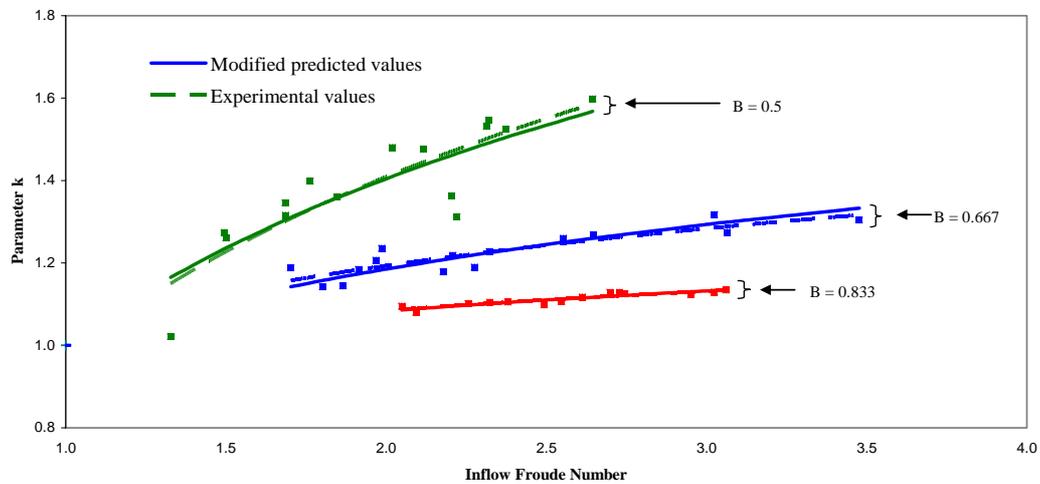


Figure 7: Comparison between measured and predicted parameter values after modification

## 5. Conclusion

The procedure for predicting the sequent depth ratio of free hydraulic jump using Equation (7) permits simple and easy application in design of enlarged stilling basin. The equation contains a parameter to account for the effect of expansion. The value of defined parameter ( $k$ ) is found to be more dependent on inflow Froude Number ( $F_1$ ) for cases of expansion ratio ( $B$ ) less than 0.667, but it is more or less independent of  $F_1$  for higher  $B$ . The experimental data, however, show fairly good agreement with the equation having modified coefficient.

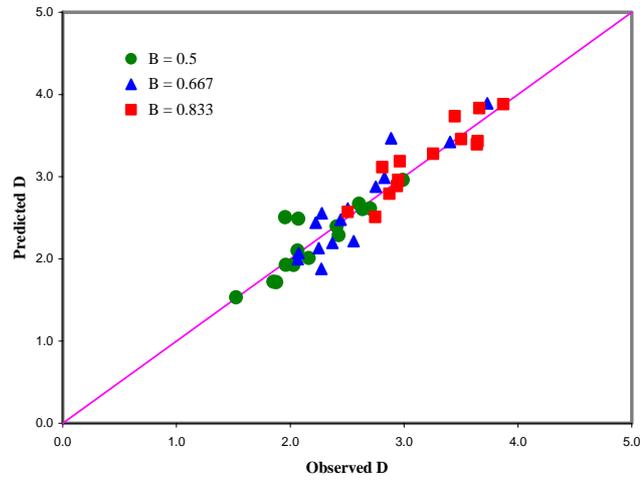


Figure 8: Predicted versus observed sequent depth ratio (D)

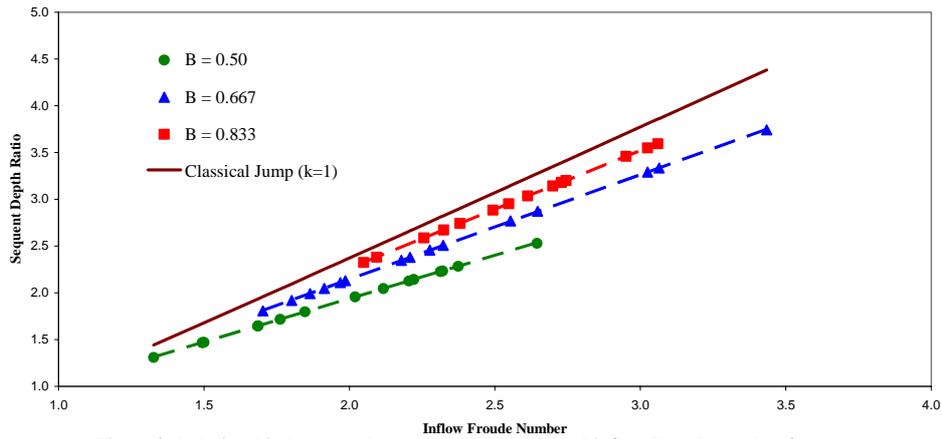


Figure 9: Relationship between the sequent depth ratio and inflow Froude number for different expansion ratios

### Notations

$b$	Channel width
$B$	Expansion ratio
$D$	Sequent depth ratio
$e$	The ratio of the flow depth at the expansion section to the upstream flow depth
$E$	Modified Froude number
$F$	Froude number
$F_p$	Pressure force
$g$	Acceleration due to gravity
$h$	Flow depth
$k$	The parameter to account for the jump position and effect of expansion ratio
$Q$	Discharge
$U$	Velocity
$\gamma$	Specific weight of water

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