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Numerical modeling of bed level changes of alluvial river

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Abstract

The Saint-Venant equations describing unsteady flow in open channels and the continuity equation for the conservation of sediment mass are numerically solved to determine the aggradation-degradation of channel bottom due to an imbalance between water flow and sediment discharge. For this purpose the MacCormack explicit finite difference scheme is used and a mathematical model is developed to predict the bed level changes of alluvial channel due to sediment over loading. To verify this numerical model, the laboratory experiments were carried out at the Hydraulics and River Engineering Laboratory of BUET. The sediment transport equation $q_s = au^b$ is calibrated through experimental runs and the values of the coefficient 'a' and 'b' were determined, which are used in the mathematical model. The computed results are compared with the experimental data. The agreement between the computed and experimental results is satisfactory.

Keywords: Bed level change, MacCormack scheme, sediment transport, laboratory experiment.

1. Introduction

Most of the river courses in Bangladesh are of alluvial nature. In fact, Bangladesh is one of the biggest Deltas in the world, which is composed of sediments carried by the three major river systems, namely, the Ganges, the Brahmaputra-Jamuna and the Meghna. The huge catchment area of these river systems supplies enormous amount of sediment to the rivers. Proper quantification of aggradation and/or degradation and changes in channel form of such alluvial rivers still has been a subject of considerable research. A systematic study on the aggradation- degradation phenomenon of these alluvial rivers of Bangladesh is, therefore, of utmost importance from the viewpoint of improved planning and design of various water development projects.

Soni et al. (1980) developed a linear diffusion model to predict the transient bed profile due to sediment overloading. Drastic simplifications made in arriving at the equation of the bed profile could not predict the transient bed profiles satisfactorily. Jain (1981) pointed out an error in the boundary condition of their analytical model. He presented the analytical solution of the aggradation problem for a more appropriate boundary condition. The analytical results were in agreement with the experimental data. Zhang and Ka11awita (1987) and Gill (1987) also developed nonlinear solution for aggradation and degradation in alluvial channels and showed better accuracy with the experimental data then the linear solutions. Gill (1987) used the method of perturbation in which the "pseudo-diffusion" coefficient was a function of the local sediment transport but was regarded a constant in the earlier linear solutions.

A number of experimental studies had been conducted to study the effect of the long term and short-term bed level changes in alluvial channels with different flow conditions. Begin et al. (1981) experimentally studied degradation of alluvial channels in response to lowering of the bed level. Soni et al. (1980) conducted an experiment that covered a wide range of flow and sediment loading conditions. They observed that after a long time, the aggradation in downstream of the section of increased sediment supply was stopped once the hydraulic conditions became compatible with the increased sediment load. Yen et al. (1989) performed a series of overloading experiments with uniform coarse sediment and found that both the aggradation wave speed and the mean sediment transport velocity increase with the initial equilibrium bed slope and with decreasing loading ratio. Series of experiments were conducted by Yen et al. (1992) to study the reversibility of an alternating aggradation-degradation process. The results show that the recovery rate decreases as standard deviation of sediment gradation increases. Alam (1998) applied MacCormack scheme to the study of aggadation-degradation in alluvial channels. Kassem and Chaudhry (2002) developed a two-dimensional numerical model to predict the time variation of bed deformation in alluvial channel bends. In this model, the depth-averaged unsteady water flow equations along with the sediment continuity equation are solved by using the Beam and Warming alternating-direction implicit scheme. Deng and Li (2003) studied the river channel equilibrium profile using one dimensional unsteady flow and sediment transport model. Yadav and Samtani (2008) developed a mathematical model using effective shear stress and bed load transport to estimate the bed load carried by the river. They computed he rate of bed load transport in weight per unit width for non-uniform bed material by collecting the field data of Tapi River, India.

In this study the physics of phenomena of changes in bed level of an alluvial river with various sediment concentrations has been investigated. The values of the coefficients ‘a’ and ‘b’ in sediment transport equation $q_s = a(u)^b$ have been determined from laboratory experimental results. Also a numerical model for studying the bed level changes in alluvial river has been developed and the model simulation has been verified with the experimental data.

2. Numerical Simulations

The successful application of a model largely depends on the correct path to develop the model for a particular problem. The assumptions used in the model development to simplify a phenomenon are sometimes crucial to the extent of their validity. Considering all these constraints, a model is developed from the basic equations describing the problem.

2.1 Governing equations

The basic one-dimensional partial differential equations describing unsteady flow in a wide rectangular channel with movable bed (Figure 1) are given by:

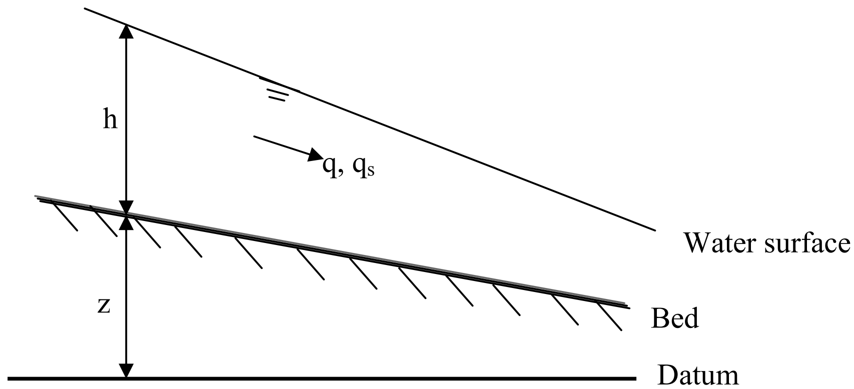

Figure 1. Definition sketch of flow variables

- Continuity equation for water

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=0 \tag{1}$$

- Momentum equation for water

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{h}+\frac{gh^2}{2}\right)+gh\frac{\partial z}{\partial x}+ghS_f=0 \tag{2}$$

- Continuity equation for sediment

$$\frac{\partial}{\partial t}\left[(1-\lambda)z+\frac{q_s h}{q}\right]+\frac{\partial q_s}{\partial x}=0 \tag{3}$$

Where,

q	=	Water discharge per unit width
h	=	Depth of flow
z	=	Bed elevation
q_s	=	Unit sediment discharge
g	=	Acceleration due to gravity
S_f	=	Friction slope
x	=	Longitudinal distance along the channel
t	=	Time
λ	=	Porosity of the bed layer

The friction slope, S_f is determined using the Manning equation (SI unit):

$$S_f=\frac{q^2n^2}{h^{3.333}} \tag{4}$$

Where, n = The Manning roughness coefficient.

The sediment discharge is estimated by an empirical power function of the flow velocity, u:

$$q_s = au^b$$

$$\text{or,} \quad q_s = a\left(\frac{q}{h}\right)^b \tag{5}$$

Where, a and b are empirical constants for which the values depend on the sediment properties.

2.2 Numerical scheme

The Eq. (1) through Eq. (3) represent nonlinear hyperbolic partial differential equations and closed form solutions of these equations are not possible except for a few simplified cases. Therefore, they are solved by numerical schemes. In this model, a finite difference scheme developed by MacCormack (1969) is used. The MacCormack scheme is a two level predictor-corrector scheme. For one- dimensional flow two alternatives of this scheme are possible. In one alternative, backward finite differences are used to approximate the spatial partial derivatives in the predictor part and forward finite differences are utilized in the corrector part. The values of the variables so determined during the predictor part are used during the corrector part. In the second alternative, forward finite differences are used in the predictor part and backward finite differences are used in the corrector part. The finite difference approximations for the derivatives of a variable 'f' are as follows:

Predictor step:

$$\frac{\partial f}{\partial t} = \frac{f_i^* - f_i^k}{\Delta t} \tag{6}$$

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^k - f_i^k}{\Delta x} \tag{7}$$

Where, i= the node in spatial grid, k = the node in time grid, Δx = the distance step and Δt = the time step. An asterisk refers to the predicted values of the variable.

Corrector step:

$$\frac{\partial f}{\partial t} = \frac{f_i^{**} - f_i^*}{\Delta t} \tag{8}$$

$$\frac{\partial f}{\partial x} = \frac{f_i^* - f_{i-1}^*}{\Delta x} \tag{9}$$

Where, the two asterisks denote the value of the variables calculated in the corrector step.

2.3 Boundary conditions

The preceding equations determine the values of h, q, and z at the new time level k+1 at every interior grid points (i = 2,3 ...N-1). The values at the boundary grid points (i = 1 and N) cannot be determined by using these equations, since there are no grid points outside the flow domain. Hence, they are determined by using boundary conditions. For one-dimensional flows, the specified interval approach of characteristic method gives acceptable results and is employed herein. Two boundary conditions at upstream boundary and one boundary condition at downstream end are required for the proper functioning of the model. In addition to these, initial conditions are needed at every computational finite difference grid point. Uniform unit discharge (q_o), uniform flow depth (h_0) and bed levels as calculated from initial bed slope (S_o) were provided as initial conditions, i.e.,

$$q(x, 0) = q_o$$
$$h(x,0)=h_o$$
$$z(x,O)=z_o \quad \text{for } x=0$$

Constant measured discharge per unit width was imposed as an upstream boundary condition. Another boundary condition came from increment of sediment discharge. However, this was not as straight forward as the former one. It was required to be translated into an equation by which the bed elevation at the upstream end could be calculated. This was achieved by assuming a fictitious node upstream from node 1 and specifying the sediment discharge at that node equal to $q_{so}+\Delta q_s$. Then, applying the backward finite difference on the spatial differential term of the sediment continuity equation, one can get the bed level at node 1 for the unknown time level k+1.

The flow depth, h at node 1 was determined by extrapolation from the already calculated values at the interior nodes using the MacCormack scheme. The downstream boundary condition was the constant depth, which was specified by $h(x_n,t) = h_o$ for $t \geq 0$. This was based on the assumption that the channel was long and the bed transients would not reach the downstream end within the period for which conditions were computed and that the variation in flow depth would be negligible. The discharge and the bed elevation at the downstream end were determined by extrapolation from the values at the interior nodes.

3. Experimental set-up and measurements

3.1 Experimental set-up

The experimental model described herein was constructed on the sand bed in the Hydraulics and River Engineering Laboratory of the Bangladesh University of Engineering and Technology, Dhaka. The experimental set-up (Figure 2) consists of two separate parts, a temporary part and a permanent part. The permanent part is the experimental facility necessary for the storage and regulation of the water circulating through the model and the guidance part. The temporary part contains the actual experimental mobile-bed model in a river. The model is built in the temporary part of the set-up. It is a mobile-bed model with fixed banks. The layout of the channel comprises of a main branch. This is a straight channel of length about 12 meters, width 1 meter and height 0.61 meter. Both of the channel banks are vertical and fixed**.** Water is flowed from the upstream reservoir to the channel through the PVC tubes (D=2.7 cm: L =30 cm) placed over the width of the entrance to get rid of larger eddies present in the water coming from the upstream reservoir. Immediately after the arrangement of such flow stabilizing tubes the sand is distributed over the width of the channel by sediment feeder. The sediment falls from the sediment feeder into the wooden structure, which distribute the sediment uniformly over the main channel width. A sediment trap (1.8 m long, 1 m wide and 1.3 m deep) is situated at the end of the branch, followed by a tail gate for the control of water levels. The permanent part acts as a facility to conduct all different types of experiment in the sand bed. The components of the permanent part are downstream reservoir, pump, pipe line, upstream Reservoir and regulating arid measuring system.

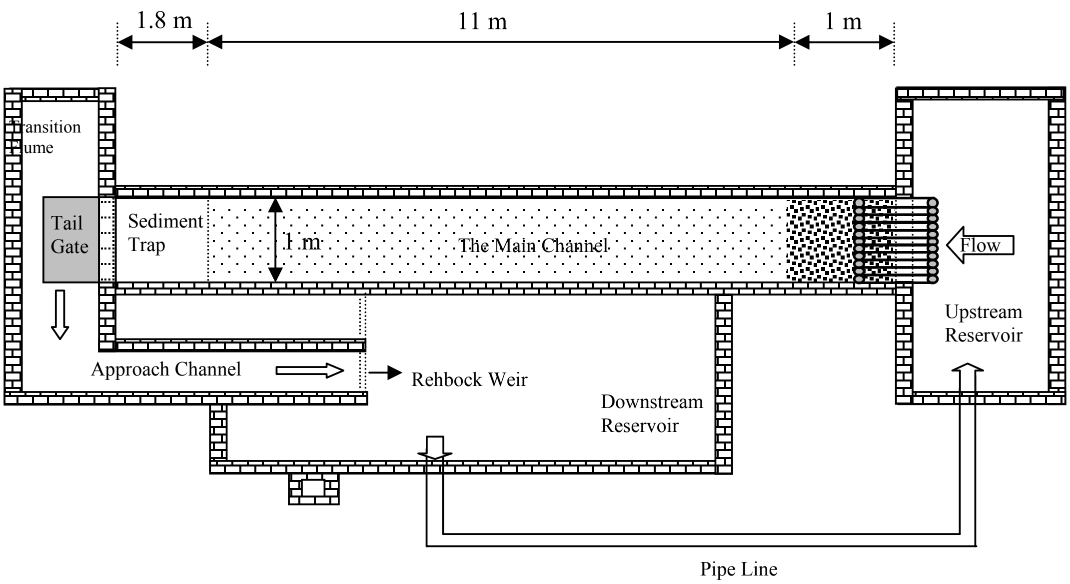

Figure 2. Layout plan of the experimental set-up

3.2 Measurements

The discharge is measured at the Rehbock weir. The water level at the crest of the weir is measured in stilling basins with point gauges, with an accuracy of 0.05 mm. The Rehbock weir can measure the discharge properly up to a discharge of 60 l/s, with accuracy (in the worst case) of 1.8%. The sediment transport rate is determined with the help of the sand trap located at the end of channel. The sand trap intercepts all sediment transported through the branch. Once a sand trap is emptied, its content is dried up and measured. The sediment transport rate is computed by dividing the amount of dry sediment by the time elapsed. The bed level was measured with a point gauge in which a special pin is used. A square plate of 2 x 2 cm^2 is fixed to the point of the pin to prevent it from sinking into the sand bed. The bed level is measured at intervals of 0.5 m, in 23 marked cross-sections at 5 points at each cross-section.

4 Data analysis, results and discussions

4.1 Determination of sediment transport constants

The sieve analysis of the sand forming the bed and injected material was done. The median diameter of the sand was 0.285 mm and the value of the coefficient of uniformity was 2. That means the sand used herein was uniformly graded. To calibrate the sediment transport equation, $q_s = a(u)^b$, 16 experimental runs were conducted with different discharges (q), and of different duration (t). After each run, sediment transported by water flow that deposited in the sand trap was collected. Then it was dried and weighted. This weight value divided by the duration of the run give the value of q_s. Finally the values of q_s and u for such 16 runs are plotted in a log-log graph paper. From this graph paper the value of a' and 'b' is calculated. The best fit line drawn through these points yielded values of 'a' and 'b' as 0.00205 and 5.25, respectively for the sediment size used in the study (Figure 3). These values of 'a' and 'b' are used in the mathematical model. The very small portion of the sediment suspended in the flow and passed over the tail gate was not measured.

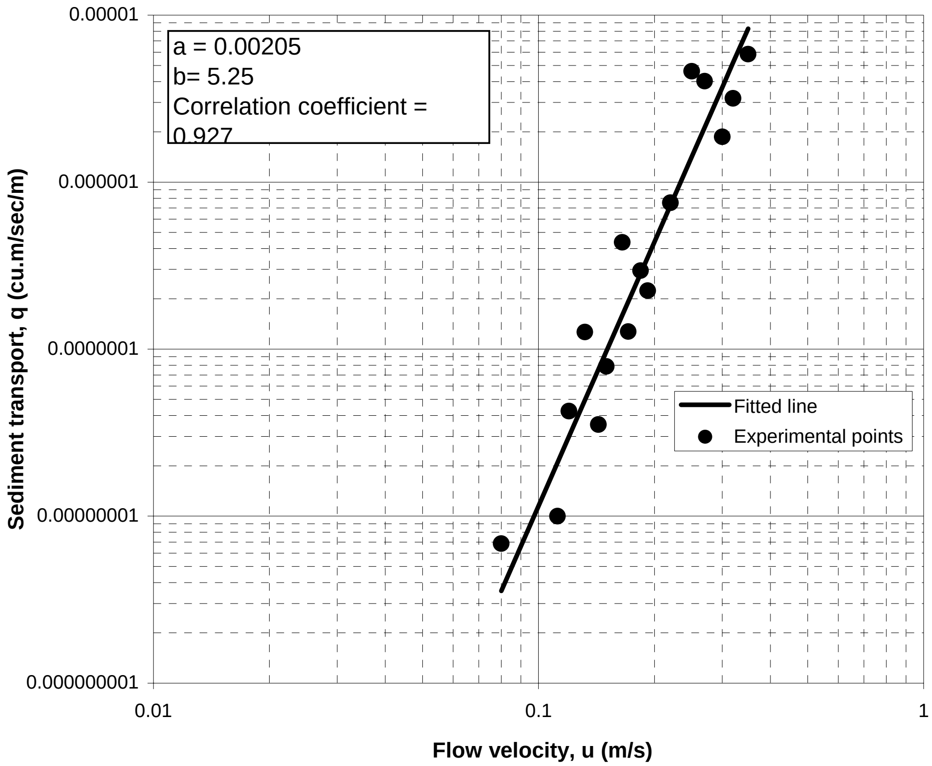

Figure 3. Determination of 'a' and ' b' for sediment transport formula

4.2 Determination of bed level changes

After the establishment of uniform flow conditions for a given discharge and slope, the sediment supply rate was increased to a predetermined value by continuously feeding excess sediment at the upstream end of the channel at a constant rate. The excess sediment load was progressively deposited in the channel. Fifteen experimental runs were carried out for different discharge (q), different bed slope (S_0) and different sediment over loading ratio. Discharge variation was within the range of 18 l/s to 20 l/s and was selected with the consideration of maximum feeding capacity of two sand feeders. The slope of the channel bed was varied from 0.00272 to 00454. The rate of sediment addition was varied from $0.5q_{s0}$ to $5q_{s0}$, where q_{s0} is the sediment transport rate for a particular run. Each run was conducted for 4 hours duration. The Manning *n* was considered as 0.022 and the porosity *p* of the sediment bed layer was taken as 0.4. The input data for experimental and numerical runs are given in Table 1.

The mathematical model was applied by dividing the channel with a total length of 11m into 22 reaches (Δx = 0.5 m). The computational time step Δt was selected according to the Courant-Friedrichs-Lewy (CFL) condition for stability, for this case it was taken to be C_n = 0.85.

Table 1
Input data for experimental and numerical runs

Run No	No. of segment	T (sec.)	Channel Length(m)	Manning's n	S_o	q (m /s)	y(m)	Sediment Ratio
1	22	14400	11	0.022	0.00272	0.02	0.07	4
2	22	14400	11	0.022	0.00272	0.02	0.072	3
3	22	14400	11	0.022	0.00272	0.02	0.068	2
4	22	14400	11	0.022	0.00272	0.02	0.71	1
5	22	14400	11	0.022	0.00272	0.02	0.07	0.5
6	22	14400	11	0.022	0.00363	0.019	0.06	4
7	22	14400	11	0.022	0.00363	0.019	0.062	3
8	22	14400	11	0.022	0.00363	0.019	0.059	2
9	22	14400	11	0.022	0.00363	0.019	0.06	1
10	22	14400	11	0.022	0.00363	0.019	0.06	0.5
11	22	14400	11	0.022	0.00454	0.018	0.05	4
12	22	14400	11	0.022	0.00454	0.018	0.052	3
13	22	14400	11	0.022	0.00454	0.018	0.05	2
14	22	14400	11	0.022	0.00454	0.018	0.049	1
15	22	14400	11	0.022	0.00454	0.018	0.05	0.5

The definition sketch of bed profile is shown in Figure 4. The equilibrium sediment transport rate under the uniform flow condition is equal to q_{s0} and the bed slope is S_0. Let the sediment supply rate at a section x = 0 be increased at a constant rate Δq_s. The bed level for $x \geq 0$ will rise with time due to increase in the sediment load. The differential equation for bed level obtained by Soni et al. (1980) is

$$\frac{\partial Z}{\partial t} = K \frac{\partial^2 Z}{\partial x^2} \tag{10}$$

Here the aggradation coefficient

$$K = \frac{1}{3}\frac{u\frac{\partial q}{\partial u}}{S_0(1-\lambda)}\left(\frac{u_0}{u}\right)^3 \tag{11}$$

where,

Z = depth of deposition at time *t* at a distance *x* from the origin
λ = the porosity of the sediment
b = the exponent in the sediment transport law
u_0 = the mean velocity under the initial uniform flow condition

Using the approximation $u \approx u_0$ and combining equation $q_s = au^b$ and the Eq. (11) and further designating the equilibrium sediment transport rate as q_{s0}, one gets the theoretical value of the aggradation coefficient, K_0, as

$$K_0 = \frac{1}{3}\frac{bq_{s0}}{S_0(1-\lambda)} \tag{12}$$

For a constant value of K, Eq. 10 can be solved for the following boundary conditions:
Z(x,0) = 0; for all x
Z(0,t) = Z_0; for all t>0
Z(x,t) = 0; for x$\rightarrow \infty$ for t>0

Substituting $\eta = x/2\sqrt{Kt}$ and $Z/Z_0 = f(\eta)$, Eq. 10 can be reduced to the ordinary differential equation: $f'' + 2\eta f' = 0$.

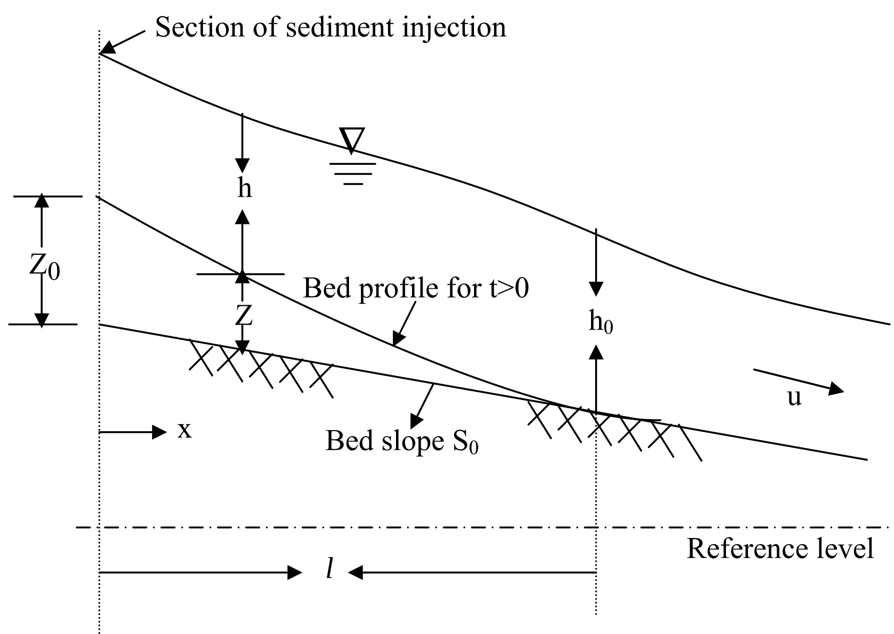

Figure 4. Definition sketch of bed profile

Two different types of plot have been produced after the successful application of the model to the laboratory study. The first type depicts transient bed surface profiles of the channel. While the other type is a dimensionless plot that shows variation of relative aggradation with distance downstream.

Figure 5 shows the comparison between the numerical and experimental results of the bed surface profiles for test Run-1 and test Run-15, having different bed slope and discharges. Each of the figures shows equilibrium and transient bed profile for 1 hr., 2 hr., 3 hr. and 4 hr. The following results are evident from these plots.

- Length of aggradation increases spatially with time.
- The depth of aggradation at a location increases with time.

These figures compare the actual data points, observed mean bed surface profiles along with computed profiles from the model. Averaging of the actual data points of these profiles was required because of the presence of ripples and dunes on the bed. The comparison with the computed profiles shows good result with the average transient profile. Close agreements are found between the experimental and computed values of the bed surface profiles.

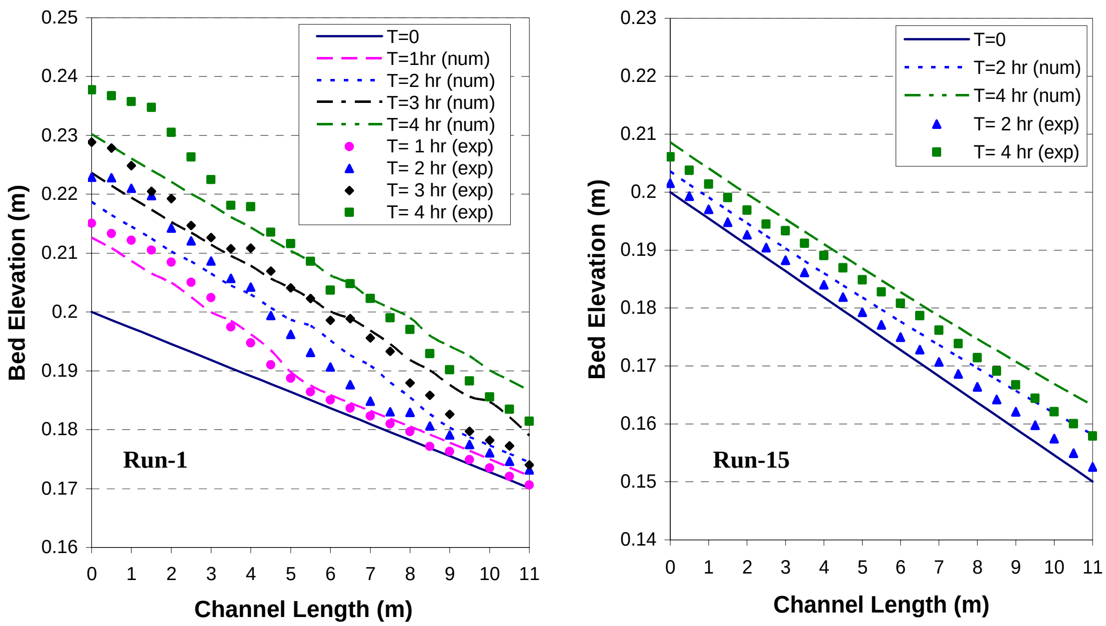

Figure 5. Comparison between experimental and numerical results of the bed surface profiles

4.3 Length of aggradation

From Figure 4 it is seen that at a given time the aggradation depth, Z, downstream of the section of increased sediment supply rate, decreases asymptotically in the longitudinal direction. However the region of asymptotic change in Z is taken to end at a section beyond which the change in Z does not exceed some prescribed value. In that case the length of aggradation, *l*, shall be a distance between the section of increased sediment supply rate and the section where the aggradation is assumed to end. Assuming aggradation ends where Z/Z_0= 0.01, a usual assumption made in problems having asymptotic behavior, e.g., as a boundary layer theory one gets [Torres and Jain (1984)]

$$l = 3.2\sqrt{K_0 t} \tag{13}$$

The non dimensional bed profile [Z/Z_0] as a function of non dimensional distance [$x/2\sqrt{(Kt)}$] for the two test runs with different bed slope and discharge (Run-6 and Run-12) are shown in Figure 6. In case of Run-12, it is seen that there are some differences between the experimental and numerical results for higher values of [$x/2\sqrt{(Kt)}$], which may happen due to shorter of channel length as explained in article 4.4 below. However, the overall performance of the model in this laboratory observation is satisfactory.

4.4 Limitation of this study

From the bed profiles it is seen that the full length of aggradation does not occur within the channel length. So the downstream boundary condition of constant water depth at the channel end is not satisfied. That is why one should use longer channel than that used herein the study.

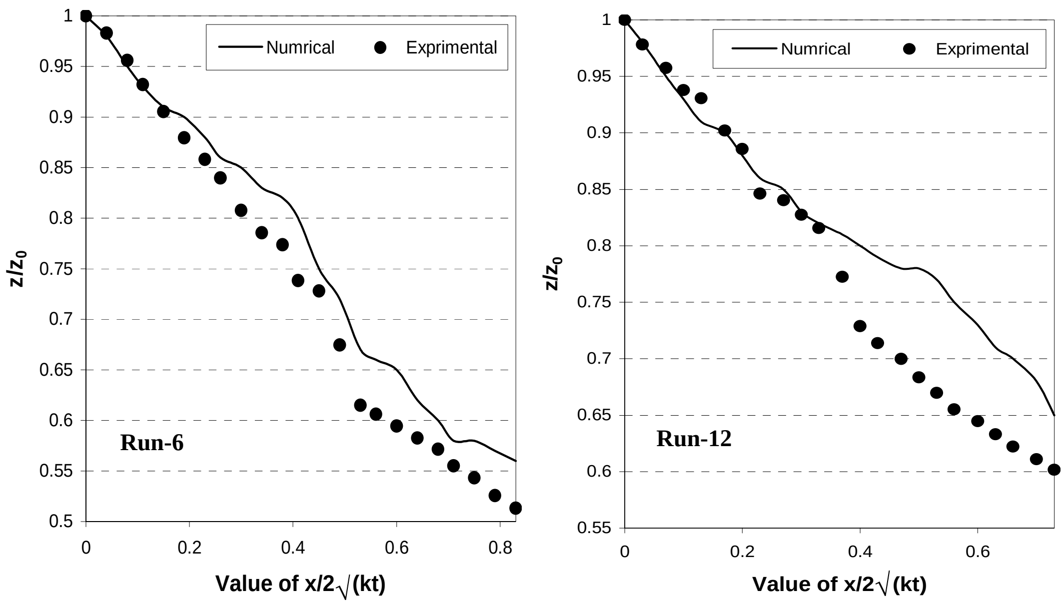

Figure 6. Comparison between experimental and numerical results of relative aggradation

5 Conclusions and recommendations

5.1 Conclusions

In this study the change in bed level of alluvial river is studied with laboratory investigation as well as application of a one-dimensional mathematical model based on MacCormack scheme. The following conclusions can be made after summarizing the present study:

- In the mathematical model, one-dimensional, unsteady, gradually varied flow equations and the sediment continuity equation are solved numerically by the second-order-accurate, explicit finite difference scheme developed by MacCormack. A simple equation for the sediment discharge as a power function of flow velocity is used.
- The numerical model was applied to simulate the bed level changes of alluvial channel due to sediment overloading. The computed results were compared with the experimental data. The agreement between them is satisfactory.
- The sediment transport equation $q_s = au^b$ is calibrated through experimental runs and the value of the coefficient 'a' and 'b' is determined, which are used in the mathematical model. The value of 'a' and 'b' were found as 0.00205 and 5.25 respectively.

5.2 Recommendations for future study

Based on the present study the following recommendations can be made for future study that may produce better results.

- This study evaluates the bed level changes due to aggradation caused by sediment over loading. The degradation study not done herein. One can use this model and study the degradation of bed level.

- The present study does not represent any particular river bed level change. It deals with laboratory cases and the relations developed herein have not been compared to real field situation. This study may be applied for a particular alluvial river to check its accuracy with real field situation.
- The MacCormack scheme can be applied to develop a two-dimensional model to study the two-dimensional problems and also a three-dimensional model to study three-dimensional problem.

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