

Multi-objective reliability based design of water distribution system

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Abstract

This paper presents a new methodology that considers the reliability based optimization of water distribution network. Three cases are considered for the assessment of reliability in water distribution networks. First case refers to the single objective optimisation of the two example networks, where minimization of cost is considered as the objective. For the second case, the problem is formulated as multi-objective with objectives: minimization of cost and minimization of break rate. And for the third case the objectives for Multi-objective optimisation are (i) minimization of cost, (ii) minimization of break rate and (iii) minimization of system reliability. The formulation is also applied to a case study. The study reveals that as break rate decreases, reliability increases. It also proves that reliability does not have direct relation with cost as seen in literatures. Results prove that multi-objective optimisation gives better results than single objective optimisation when reliability of the network is considered.

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1. Introduction

The investments and maintenance costs associated with implementation of a new water distribution system as well as rehabilitation or expansion of existing water distribution systems are very high. Most of the optimisation studies done earlier related mostly to the single objective function of minimising cost subjected to constraints. An important issue to be considered is maximization of reliability of the system, thereby changing the problem into a multi-objective one. Reliability of water distribution networks relates to two types of failure, mechanical failure of system components and hydraulic failure caused by changes in demand and pressure head (Tabesh et al. 2009). Pipe failures in urban water distribution networks lead to financial and capital losses for repair and reduce the reliability of the network due to lowering of the pressure or due to interruption of the water supply in parts of the distribution network, which ultimately

leads to dissatisfaction of customers. Sensitive customers such as industrial centers, governmental buildings, hospitals, etc., are affected.

Since the design of water supply and distribution systems involve a number of parameters, any uncertainty in selecting their values would affect the performance and hence the reliability of the system. The design procedures of distribution systems should consider the future scenarios too. Breaking of pipe is one of the important parameters to be considered in distribution system design. Generally, small-diameter pipes are more liable to breaks than large-diameter pipes (Kettler & Goulter 1985). An initial cost-optimisation model that does not consider the pipe breaks during the lifetime may result in pipes of smaller diameter. This might prove to be less optimal over a long run, as the distribution system tends to experience more breaks when compared to a system with slightly larger diameter pipes.

Tung(1985) evaluated reliability using “minimum cut set method”. Minimum cut set is a set of system components that when failed cause failure of the system. Reliability is defined as the probability that the system performs within specified limits for a given period of time (Mays et al,1987). Network reliability is often considered as the extent to which the network can meet customer demands at adequate pressure under normal and abnormal operating conditions. From the literature review it is found that minimum cut set method is more efficient for reliability analysis of water distribution network. So in this study reliability is calculated using minimum cut set method. The main objectives of the study are: single objective optimisation of two example networks, (Two loop network and Hanoi network) multi-objective optimisation of the networks including break rate as the second measure, multi objective optimisation of the same networks including reliability as the third objective. And finally these formulations are applied to a case study. The area selected for the study is Peroorkada zone of Trivandrum water supply scheme, Kerala state.

2. Methodology

2.1 Case 1: Single objective optimization

Single objective optimisation is done using Genetic Algorithm in GANetXL which is an add-in for Microsoft Excel. To enable simulation of the network EPANET is linked with GANetXL. Single objective optimisation mainly aims to minimize the network cost. In this study material cost of different diameter pipes are considered. The objective is:

1. Minimize the cost of the network

$$\text{Minimize } f_1 = \sum_{p=1}^{np} C_p L_p \quad (1)$$

where C_p is the cost in units and L_p is the length in meters

Subjected to constraints

$$(H_{avl}-H) \geq H_{min} \quad (2)$$

where H_{avl} is the head available at a node and H_{min} is the minimum head required at the same node. H is elevation corresponding to that node.

2.2 Case 2: Multi-objective optimization (Two Objectives)

Non-dominated Sorting Genetic Algorithm-II (NSGA-II) introduced by Deb et al. is an extension to an earlier Multi-Objective EA called NSGA, incorporates elitism to maintain the solutions of the best front found. NSGA-II is used for the multi objective optimisation of the networks. The second case is based on the minimization of network cost and minimization of break rate. Second objective is to account for mechanical uncertainties, while the constraint is to account for the demand uncertainty. The variation in number of breaks with respect to time is more useful for analysis over long periods. Shamir and Howard(1979) proposed two expressions, based on linear and exponential variation of break rates with time, which is as follows:

$$N(t)=N(t_0) + A(t-t_0) \quad (3)$$

$$N(t)= N(t_0) \exp A(t-t_0) \quad (4)$$

where $N(t_0)$ is the number of breaks per year per unit length at unit time $t= t_0$ varies from 0.033 to 0.656. $N(t)$ is the number of breaks per year per unit length at unit time t , A is the break rate coefficient which varies from 0.01 to 0.15.

In the present study, the exponential expression is considered for calculating number of breaks per year per unit length. And t_0 is taken as zero years. The break rate coefficient is taken as 0.01 for the entire pipes. The pipe diameter having initial break rate less than 0.033 (least value of initial break rate given by Shamir and Howard(1979)) is selected for the pipe which carries water from reservoir to the network. All other pipes are selected based on average value of the minimum and maximum values of the initial break rate. Then the break rate after 5 years, 20 years and 50 years are found by using Eq.(4). The formulated multiple objectives functions are given below:

1. Minimize the cost of the network
2. Minimize $f_1 = \sum_{p=1}^{np} C_p L_p$ (5)

where C_p is the cost in units and L_p is the length in meters

3. Minimize the break rate of the pipes

$$\text{Minimize } f_2 = \sum_{p=1}^{np} (N_{(t_0)} + N_{(t_0)} \exp^{A(t-t_0)}) \quad (6)$$

where $N(t_0)$ is the number of breaks per year per unit length at unit time $t= t_0$, $N(t)$ is the number of breaks per year per unit length at unit time t , A is the break rate-coefficient.

Subjected to constraints

$$(H_{avl}-H) \geq H_{min} \quad (7)$$

where H_{avl} is the head available at a node and H_{min} is the minimum head required at the same node. H -is the elevation corresponding to that node

2.3 Case 3: Multi-objective optimization (Three Objectives)

This case considers three objectives for optimisation. For improving the performance of the system, a program code is developed in Visual Basic language based on minimum cut set method for finding the reliability of the network and set as the third objective in GANetXL software. In this study only single pipe failure is considered for the reliability analysis. The objectives considered are:

1. Minimize the cost of the network

$$\text{Minimize } f_1 = \sum_{p=1}^{np} C_p L_p \quad (8)$$

where C_p is the cost in units and L_p is the length in meters

2. Minimize the break rate of the pipes

$$\text{Minimize } f_2 = \sum_{p=1}^{np} (N_{(t_0)} + N_{(t_0)} \exp^{A(t-t_0)}) \quad (9)$$

where $N(t_0)$ is the number of breaks per year per unit length at unit time $t = t_0$, $N(t)$ is the number of breaks per year per unit length at unit time t , A is the break rate coefficient.

3. Maximize reliability of the system

$$\text{Maximize } f_3 = R_s \quad (10)$$

where R_s is the system reliability

Subjected to constraints

$$(H_{avl} - H) \geq H_{min} \quad (11)$$

where H_{avl} is the head available at a node and H_{min} is the minimum head required at the same node. H is the elevation corresponding to that node.

Goulter and Coals (1986) developed a method to determine the probability of failure of individual pipes. The probability of failure of pipe j , P_j can be determined using the Poisson probability distribution

$$P_j = 1 - e^{-\beta_j} \quad (12)$$

and

$$\beta_j = r_j L_j \quad (13)$$

where β_j = expected number of failures per year for pipe j , r_j is the expected number of failures per year per unit length of pipe j .

Using failure data obtained from the City of St. Louis (1985), a regression equation was developed to compute the expected number of failures, r_j (breaks / mile / year), as

$$r_j = 0.819 e^{-0.1363 D} \quad (14)$$

where D is the pipe diameter in inches.

If there are n components (i pipes) in the ith minimum cut set of a water distribution network, failure probability of the jth component in the ith minimum cut set, P_{ij} is

$$P_{ij} = 1 - e^{-\beta_j} \quad (15)$$

Failure probability of ith minimum cut-set is

$$P(\text{Mu}) = \prod_{j=1}^{N_i} P_{ij} = P_{i.1} \cdot P_{i.2} \cdot P_{i.3} \dots P_{i.n} \quad (16)$$

The basic assumption is that the occurrences of the failure of the components within a minimum cut-set are statistically independent. *e.g.* If a water distribution network has 4 cut sets, MC₁, MC₂, MC₃, and MC₄ for the system reliability, the failure probability of the system, P_s is defined as

$$P_s = P(\text{MC}_1 \cup \text{MC}_2 \cup \text{MC}_3 \cup \text{MC}_4) \quad (17)$$

And it is mathematically proved by Ross (1985) as

$$P_s = \sum_{i=1}^M P(\text{MC}_i) \quad (18)$$

A flowchart showing reliability analysis using minimum cut-set method is shown below Fig.1.

where M is the no. of minimum cut sets in the system. Then the system reliability R_s,

$$\begin{aligned} R_s &= 1 - P_s \\ &= 1 - \sum_{i=1}^M P(\text{MC}_i) \end{aligned} \quad (19)$$

Same approach is used for finding nodal reliabilities also.

3. Study on two example networks

A. Example network 1

The example network considered is a Two loop network consisting of 8 pipes and 6 nodes is shown in Fig. 2. The minimum head required at each node is 30 m. For the optimisation, a set of diameters are selected. The same diameters are used for Hanoi network also. The diameter –cost details are shown in Table 1.

B. Example Network 2

The second example network considered is Hanoi network consisting of 34 pipes and 32 nodes and three loops. The minimum head required at each node is 30 m. And it is shown in Fig. 3.

4. Results and Discussion

A. Results of single objective optimisation-Case I

Parametric study has been done for fixing the GA parameters. Each parameter considered that is, population, Cross over, number of generations, mutation, selector and algorithm are varied keeping all other parameters constant.

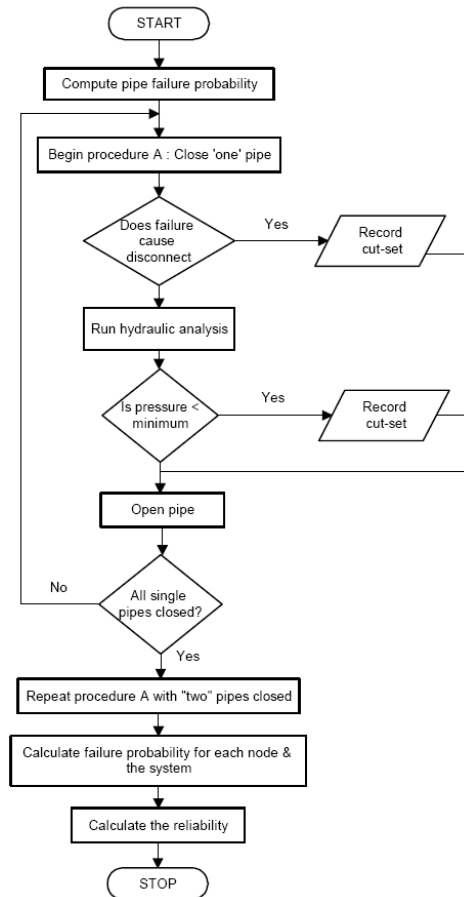


Figure 1. Minimum cut-set reliability flow chart

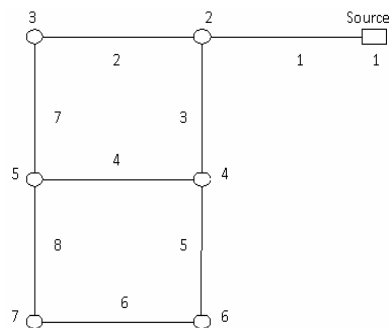


Figure 2. Two loop network

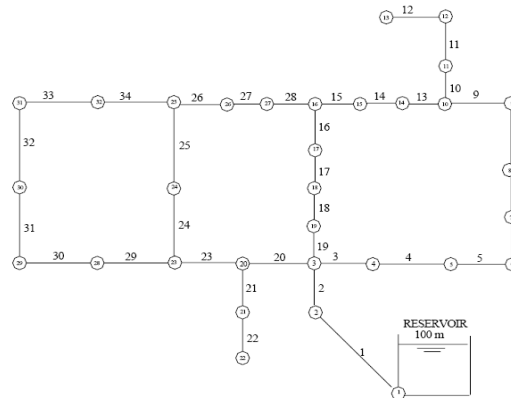


Figure 3. Hanoi network

Table 1
Diameter –cost details

Diameter (mm)	25.4	50.8	76.2	101.6	152.4	203.2	254	304.8	355.6
Cost (units)	2	5	8	11	16	23	32	50	60

Table 1 (continued)
Diameter –cost details

Diameter (mm)	406.4	457.2	508	558.8	609.6	762	1016
Cost (units)	90	130	170	300	550	731.69	1126.51

The results obtained from the parametric study for both the networks are listed in Table 2. Using these parameters the single objective optimisation is done. For Two loop network, 200 solutions are obtained, the reliability of each such set of solution is calculated using minimum Cut-set method in which only 42 solutions are found to be reliable.

Table 2
Parameters selected for the multi-objective optimization

Parameters	Two looped network	Hanoi network
Population	200	200
Cross over rate	0.95	0.95
No. of generations	500	500
Mutation	0.06	0.013
Selector	Roulette by rank	Tournament

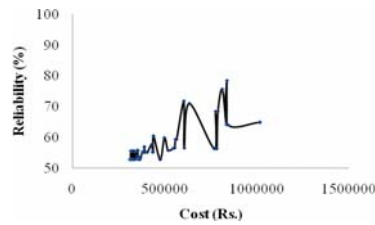


Figure 4. Cost-reliability relation for single objective optimisation (Two loop network)

From Fig. 4, it can be seen that reliability is independent of cost. The same reliability can be achieved with different diameter combinations, for example, 55% reliability can be obtained with 16, 17, 19, 20 and 22nd solution. And also with same cost different reliability is possible, for example, 11th set has a reliability of 54% and 12th set has a reliability of 56%, but both the sets have a cost of 3.56 lakhs. The total cost varies from 3.09 lakhs to 10.18 lakhs. The maximum value of reliability obtained for Two loop network is 79%. Single objective optimisation of Hanoi network yielded only 10 solutions which are reliable. Since in this case cost is the important factor, the reliable solutions are less and this is the major draw back of least cost design of water distribution system. For Hanoi network maximum reliability obtained is 78% corresponding to a cost of 235.58 lakhs and the cost obtained varies from 213.95 to 261.61 lakhs.

B. Results of multi-objective optimisation-Case II

Multi objective optimisation is done with the two objectives for both the network with the previously fixed parameters. The system reliability of the network is found by minimum cut set method. For the Two loop network among the 200 solutions, the reliable solutions plotted are shown in Fig.6. The maximum value of reliability obtained for Two loop network for the second case is 97%. In Fig.6, there is a sudden reduction in reliability even though the cost is high or pipe diameter is large which indicates that by choosing larger diameters alone, we cannot improve reliability. Fig.7 indicates that for different solutions with different cost and reliability same break rate is possible. For example, for the solution no. 32 and 33 the costs are 6.8 lakhs and 6.9 lakhs respectively and reliabilities are 93% and 92% respectively but the break rates are same, i.e. 1.7969. The multi-objective optimisation results of Hanoi network is shown in Fig.8 and 9.

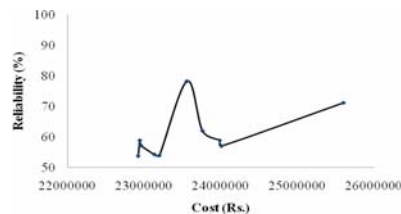


Figure 5. Cost-reliability relation for single objective optimisation (Hanoi network)

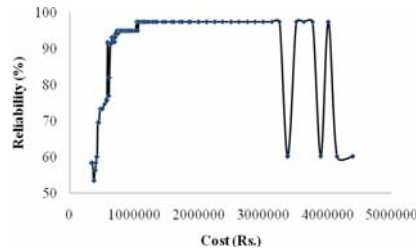


Figure 6. Variation of reliability with cost (Two loop network)

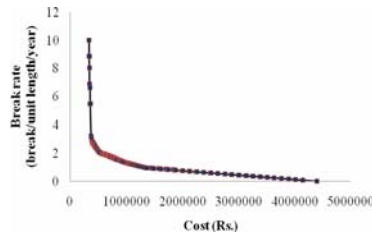


Figure 7. Variation of break rate with cost (Two loop network)

From the figures it is seen that for Hanoi network, in the case of multi objective optimisation (two objectives), the maximum reliability is 80% which is more comparable with to the previous result of single objective optimisation.

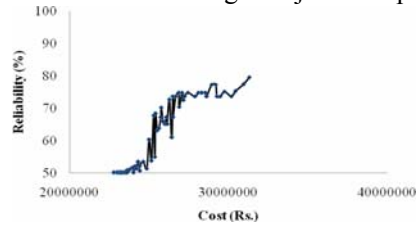


Figure 8. Variation of reliability with cost (Hanoi network)

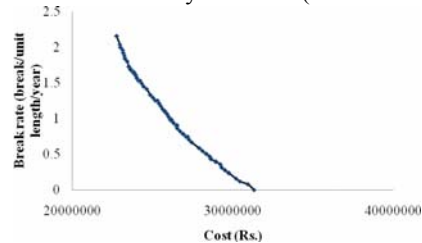


Figure 9. Variation of break rate with cost (Hanoi network)

C. Results of multi objective optimization –Case III

The multi objective optimization of the network with three objectives is done. Among the 200 solutions obtained, 197 solutions are reliable. The variation of reliability with cost is shown in Fig.9. It is clear that the reliability for Two loop network has increased to 100%.

The variation of break rate with cost is also shown in Fig. 10. Break rate varies depends on the pipe diameter, but for the solutions which have same reliability as well as same cost have different break rates, for example solution no. 7 and 8, both the solutions have

same reliability, same cost but different break rates. The maximum reliability obtained for Hanoi network in third case is 98%.

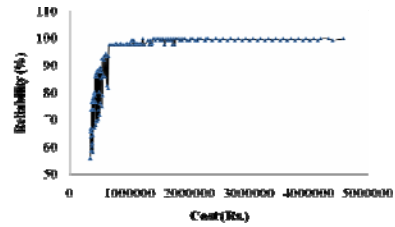


Figure 10. Variation of reliability with cost (two loop network)

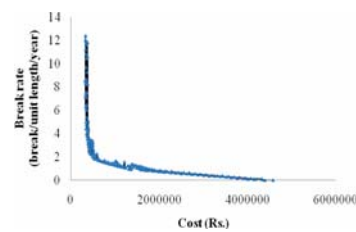


Figure 11. Variation of break rate with cost (two loop network)

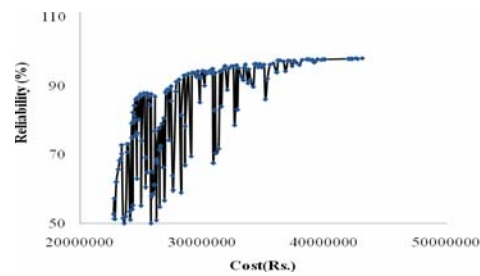


Figure 12. Variation of reliability with cost (Hanoi network)

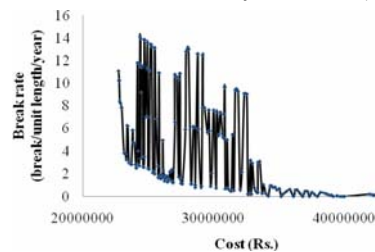


Figure 13. Variation of break rate with cost (Hanoi network)

D. Application to the case study

In Trivandrum water supply scheme, the city is divided into four zones for easiest supply of water to consumers. They are Peroorkada, Thirumala, Central Zone and Low level Zone respectively. Out of these zones, Peroorkada zone has been chosen in the present study. Peroorkada zone consists of eleven wards, Medical College, Palayam, Kanjirampara, Nanthancode, Kunnukuzhy, Pattom, Kesavadasapuram, Kannammoola, Kowdiar, and Sasthamangalam. The location map of this zone is shown in Fig. 14. The

ground is sloping towards west and elevated places are seen in the wards located on the north west, such as Kanjirampara, Sasthamangalam, Kowdiar and Kuravankonam.

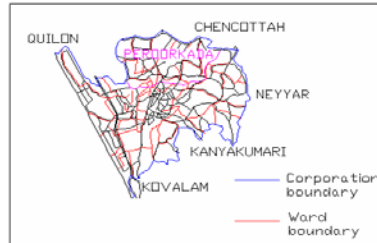


Figure 14. Location map of Peroorkada zone

The study area selected comprises an area of 15.46 km². Total population as per 1991 census is 137714. Total water demand is 25.82m³/min. There is only one reservoir at Peroorkada which has a capacity of 8 million liters. The total domestic connection is 13700, non-domestic connection is 180 and industrial connection is around 6. The number of house-hold connection is approximately 18470. The minimum head required at each node is 8m. The Peroorkada network consists of 99 nodes and 114 pipes. Since it is a very old network the roughness values are not the same as the stage at which it is installed. The aging of pipe causes fouling formation in pipe which increases the roughness and reduces the flow velocity, pressure, inner area of the pipe and in turn flow through the pipe. The roughness after time *t* years is given by Vassiljev and Koppel(2006)

$$CHW_i(t) = 18.0 - 37.2 \log \left(\frac{e_{0i} + a_i(t + g_i)}{D_i} \right) \tag{20}$$

where $CHW_i(t)$ is the Hazen Williams friction coefficient in pipe *i* at year *t*, e_{0i} is the initial roughness of pipe in metre, a_i roughness growth rate in feet per year, D_i is the diameter of pipe in feet, g_i age of pipe *i* at the present time, *t* is the time elapsed from present to future periods in year.

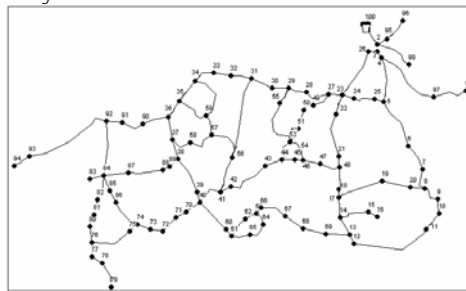


Figure 15. Water Distribution Network Map of Peroorkada zone

Only the first and second cases are applied to peroorkada zone since in the third case the computing time taken is relatively large.(34 hours).

Results: Case1 resulted in 200 unique solutions but none of the solution is reliable. Case 1 is repeated for a minimum head of 20m, 30m, 40m etc. Upto 30m the solutions obtained were unreliable solutions and when the head increased greater than 30m,

reliable solutions were obtained but the cost of the network increased to a very high value. But, the second case gave reliable results at 10m minimum head and the results are shown in Fig. 16. The maximum reliability obtained for peroorkada network is 78%. The relation between cost and break rate for peroorkada zone is shown in Fig. 17.

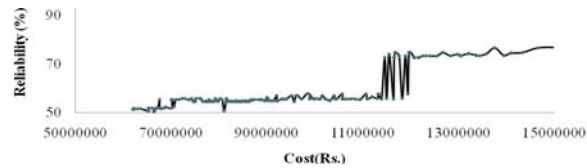


Figure 16 Variation of reliability with cost for Peroorkada zone

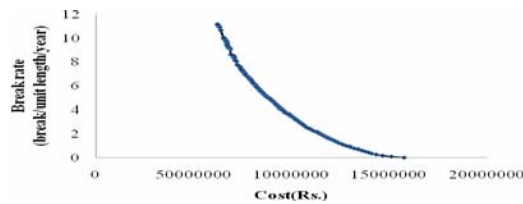


Figure 17. Variation of break rate with cost for Peroorkada zone

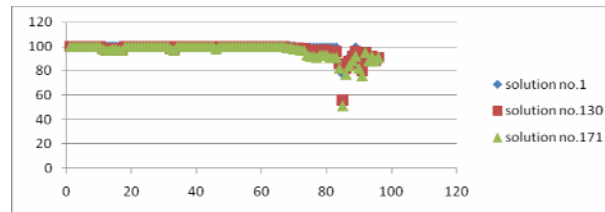


Figure 18. Nodal reliability at different nodes for Peroorkada zone

Three solutions with zero, maximum and average value of break rate is selected and the nodal reliability obtained at each node is shown in fig. 18. From fig. 18, it is found that some nodes are critical nodes i.e., nodal reliability and head available at these nodes are less compared to other nodes. The critical nodes identified are nodes 86, 87 and 88 which are near Medical College, node 92- Ulloor, node 93-Akkulam, and node 97-Mannammoola.

5. Conclusions

Single objective and multi objective formulations are applied to example networks and case study. The reliability obtained for the two example networks based on multi objective optimisation, is found to be higher compared to single objective optimisation. Also when break rate is considered, reliability of the solution is increased further. For the first case, reliability of the two loop network is only 79%, whereas in the third case, it is relatively large and increased to

100%. For Hanoi network the reliability has increased from 78% in first case to 98% in the third case.

In the case of Peroorkada network reliability has increased from 0% to 78%. The critical nodes are identified and they are, node 86,87 and 88 which are near Medical College, node 92- Ulloor, node 93-Akkulam, node 97-Mannammoola. The nodal reliability at the critical nodes 97 and 93 are low. This is because the nodes are dead ends and this portion does not form part of loops. If the nodes are part of loops, any breaking of intermediate connecting pipes may not affect the supply at these nodes. By providing an additional pipe between nodes 93 and 83 at Akkulam area and a pipe between 97 and 5 near the source, the reliability at these nodes can be improved. Hence, for long term design initial expenditure may be higher due to selection of large diameter pipes, but maintenance and repair cost may be reduced in future. So considering entire design period, this is found to be more economical. Therefore realistic design of water distribution network needs to be dealt as a multi-objective optimization problem.

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