

Effect of span length on the dynamic amplification factor in the deck of concrete box girder bridges

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Abstract

This paper presents an investigation of the dynamic responses on the vertical deflections and transverse stresses of the cantilever slab of a concrete box girder bridge subjected to moving traffic loads. A finite element model to analyze a three-span concrete box girder bridge is developed. A vehicle is simulated according to the HS20-44 truck design loading of AASHTO specifications and moving vehicle load is applied at various points on the cantilever part of the span along the vehicle trajectory. Full transient dynamic time history analysis is performed. The dynamic deformation and stress responses of the bridge are evaluated for different span length in a systematic approach. Span length is considered as an important parameter that influences bridge-vehicle interaction. Dynamic Amplification Factors (DAF) are evaluated at various points along the central span in terms of deflections at the tip and transverse stresses at the root of the cantilever slab. The DAFs are obtained by comparing the stress and deflection values found from dynamic analysis to the same obtained from static analysis. It has been found that the DAF is critical at the piers of the bridge. The effects of span length on DAF were found to be significant. Based on the overall findings, a Dynamic Amplification Factor of 3.0 and 4.0 can be recommended over static results for vertical deflections and transverse stresses respectively on the bridge deck.

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Keywords: Dynamic Amplification Factor, box girder, vehicle load, transverse stress

1. Introduction

In recent years, from some practical observations, it has been acknowledged that only static analysis is inadequate for ensuring the safety of a structure. It is essential to determine the responses of a structure due to dynamic disturbance as well as those due to static disturbance. In safety assessments, the dynamic effect caused by the vehicle on a bridge deck is taken into consideration using equivalent loads determined by multiplying the static loads by a Dynamic Amplification Factor (DAF). In case of highway and

railway bridges, improper exercise of DAFs can lead to risky road conditions for vehicular movement. Consequently, the practice of inappropriate DAFs can have remarkable financial implications as well.

The dynamic response of bridges due to vehicular loading has been a subject of interest to engineers and bridge designers for more than hundreds of years. Over the last few decades, a significant amount of research has been conducted in the area of bridge dynamics. These researches have been both analytical and experimental in scope. As the dynamic response of bridges is complicated by a number of independent but interacting factors, different studies have produced conflicting results and conclusions.

The Dynamic Amplification Factors defined in codes have classically been derived from the measurement or simulation of global traffic action effects in the main structural elements of bridges. On the other hand, local dynamic effects in deck slabs have not yet been studied in detail apart from few approaches based on the fundamental theory of plate vibration: [e.g. ASCE (1931), Gupta and Trail-Nash (1980), Agarwal and Billing (1990), Green and Cebon (1994)]. References are rare dealing with deck slabs in particular.

An ASCE committee (ASCE 1931) reported on 'Search for available data on the subject of impact in highway bridges'. One of the observations of the committee was that existing data were too meager to establish a relationship between impact and span length. Approximately the same impacts were indicated for all spans.

Billing (1984) and Billing and Green (1984) conducted tests on 27 bridges to obtain comprehensive data to support the Ontario Highway Bridge Design Code (OHBDC 1979). Accelerations, deflections and strain measurements were made to characterize the dynamic response of the bridges. They concluded that mean dynamic amplifications were relatively modest, even though for some tests dynamic amplifications greater than 0.5 were observed. Inbanathan and Wieland (1987) performed an analytical investigation of the dynamic response of a simply supported box girder bridge due to a vehicle moving across the span and found the effect of vehicle mass on bridge response to be more significant in high speeds. They concluded that dynamic stresses developed by a heavy vehicle moving over a rough deck at high speeds were larger than those predicted by several bridge codes. Coussy et al. (1989) presented an analytical study of the influence of random surface irregularities on the dynamic response of bridges. The authors concluded that the DAF decreases with span length but not as strongly as given by the codes of different countries. The investigation also showed that, in the absence of significant surface irregularities, the dynamic amplification factor is independent of bridge span. Hwang and Nowak (1989, 1991) and Nowak et al. (1990) developed procedures for calculating the statistical parameters of dynamic loads on highway bridges to be used in development of a reliability-based bridge design code. Variables considered included different vehicle characteristics (mass, suspensions and tires, axle arrangement and speed), road roughness and bridge properties (span, mass, support type, material and geometry). Based on the results, a uniform dynamic load factor of 0.25 was recommended for all spans greater than 6 m.

Wang et al. (1996) studied the free vibration characteristics and the dynamic response to a multivehicle load moving across rough bridge deck of continuous and cantilever thin-walled box girder bridges and concluded that the most important factor which affects the impact of cantilever bridges is the vehicle speed.

Broquet et al. (2004) performed a parametric study, based on the stimulation of bridge-vehicle interaction, to investigate characteristic properties of the dynamic behavior of the bridge deck slabs of concrete bridges and to deduce the distribution of DAFs throughout the deck slab. The variation of DAF was analyzed for various factors i.e. vehicle speed, vehicle mass, road surface condition, bridge cross section. The dynamic amplification factor varied between 1.0 and 1.55 for the bridges and vehicles studied. They found the DAFs to increase with road roughness and decrease with the vehicle mass. They concluded that DAFs in deck slabs are not influenced significantly by vehicle speed, location in a deck slab, type of superstructure, slab geometry, slab boundary conditions or slab stiffness. A uniform dynamic load factor of 0.25 was recommended for all spans greater than 6 m. Brady et al. (2006) used relatively simple numerical models to investigate the effect of vehicle velocity on a bridge's dynamic amplification. A set of critical velocities were determined associated with peaks of dynamic amplification for all beams. Shi et al. (2008) calculated the responses of vehicle induced dynamic bridge by modeling the bridge and vehicle as one coupled system. A parametric study was conducted to analyze the effects of different truck speeds and different road surface conditions. The faulting condition of the approach slab was found to cause significantly large dynamic responses in short-span slab bridges.

Very recently cracks were observed along the root of the cantilever part of the deck of Jamuna Multipurpose Bridge of Bangladesh. The reasons behind the formation of cracks might be high impact in the transverse direction and low amount of required reinforcement specified by the conventional theories. Researchers came to predict the requirement of steel in transverse direction in order to minimize the possibility of development of cracks. Therefore, it is necessary to determine the dynamic Amplification Factor for tip vertical deflections and transverse stresses at the root of the cantilever part of the deck slab.

Although some good researches as discussed above can be found in overall bridge dynamics, very few researches have been conducted related to the DAF in the cantilever part of deck slabs of bridges for transverse stresses and vertical deflections. Most of those involved the study of local dynamic effect in the longitudinal direction. For certain bridge types e.g. box girder bridge, cantilever part of the deck slab has immense structural and functional importance. The previously performed studies did not take into account the effect of vehicular loading on the stresses in the transverse direction of the cantilever part. No defined rule was mentioned in AASHTO Standard Specifications for Highway Bridge (2002) pertinent to local dynamic effect in the transverse direction of the cantilever part of deck slabs.

The major objective of this paper has been to enlarge our knowledge about different parameters related to responses due to static and dynamic loading conditions along with the DAF. In this paper, analysis was performed to get the DAF for the bending stresses produced in the transverse direction of the cantilever of box girder bridge due to vertical deflection at the tip of the cantilever with respect to different span lengths.

2. Finite Element Modelling

With the advent of diversified and flexible numerical tools like finite element method, it has become possible to model bridges for both static and dynamic analysis. Therefore, a finite element model has been modeled for determining the dynamic response of the deck of a box girder bridge. In this chapter, the actual work for the development of finite element model has been described. Bridge modeling, vehicle modeling, and vehicle

speed simulation with proper boundary and loading conditions have been described. Dynamic impact has been realized by measuring Dynamic Amplification Factor (DAF). The DAFs have been obtained by determining responses with respect to both static and dynamic loading with the help of finite element model.

2.1 Model for Finite Element Analysis

In this paper, a typical three-span box girder bridge has been selected as the model for finite element investigation. Moving standard vehicle loads have been applied on the cantilever part of the bridge and both static and full transient dynamic analyses have been performed.

2.1.1 Geometries of the substructure of the parametric study

Side elevation of the reference model is shown in Figure 1 and typical geometries of the box section are illustrated in Figure 2. A specific and comprehensive parametric study has been executed in this study. The key parameters considered have been vehicle speed, width of the cantilever, wheel position and thickness of the slab. The reference (median) geometries of these key parameters of the model have been taken for width of the cantilever slab as 4.0 meter, for wheel position from the root of the cantilever as 1.0 meter, for maximum and minimum thickness of cantilever slab as 400 mm and 200 mm respectively and for vehicle speed as 100 km/h. The same cross-section has been maintained throughout the bridge.

To avoid complexities and to keep the number of nodes within the limit, approximation has been made by modeling the cantilever part for uniform thickness for half of the width and modeling the other half with a reduced uniform thickness. Similarly, the deck slab within the box has also been modeled with two separate thicknesses as shown in Figure 3.

2.1.2 Load

In this paper, HS20-44 highway standard design truck of AASHTO (2002) has been taken as the highway truck loading. The vehicle has been idealized as concentrated forces moving along the deck with its trajectory parallel to the edge of the cantilever. The vehicle has been assumed to move at a constant velocity and the mass of the vehicle has not been considered to be applied on the deck.

2.1.3 Boundary condition

The bridge has been modeled as simply supported at pier locations. Diaphragms have been provided at all piers of the bridge.

2.1.4 Materials

In this investigation, material used for the model was considered linear and isotropic. The structure has been assumed to be made of reinforced concrete. Properties of the materials are taken as Poisson's ratio, $\mu = 0.15$, modulus of elasticity, $E = 20 \times 10^6$ kN/m² and density, $\rho = 2.44 \times 10^{-6}$ kN/m³. However, Poisson's ratio of the material has very little effect on this study.

2.1.5 Damping characteristics

Effects of damping parameters are insignificant for analyzing dynamic responses. For this reason, damping parameter has been taken as zero for conducting the analysis.

2.1.6 study parameters

Parametric study is an important part of this present investigation. For determining dynamic response of the deck slab, several parameters have been chosen i.e. span length, slab thickness, slab width, wheel position, vehicle speed etc. In this paper, all the other parameters except span length have been taken as constants. The aim has been to identify

influence of span length on the dynamic response as well as to determine the corresponding maximum DAFs. Different values of each parameter are shown in Table 1.

Table 1
Geometries of the box girder bridge in parametric study

| Parameters | Unit | Geometry |
|---|------|--------------------------------|
| Thickness of the Thicker Portion of the Slab, T_1 | mm | 300, 400, 500 |
| Thickness of the Thinner Portion of the Slab, T_2 | mm | 150, 200, 250 |
| Width of the Cantilever, B | m | 3, 3.5, 4, 4.5, 5 |
| Wheel Position, C | m | 1 |
| Vehicle Speed, V | km/h | 100 |
| Span Length, L | m | 25, 50, 75, 100, 125, 150, 175 |

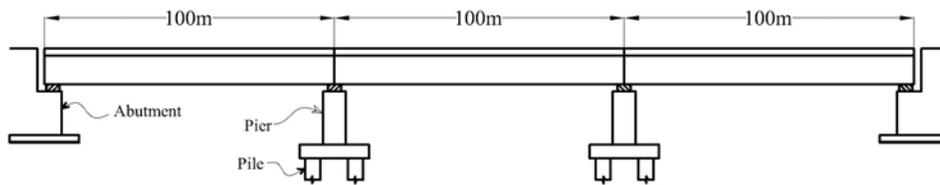


Figure 1. Conceptual side elevation of the reference model

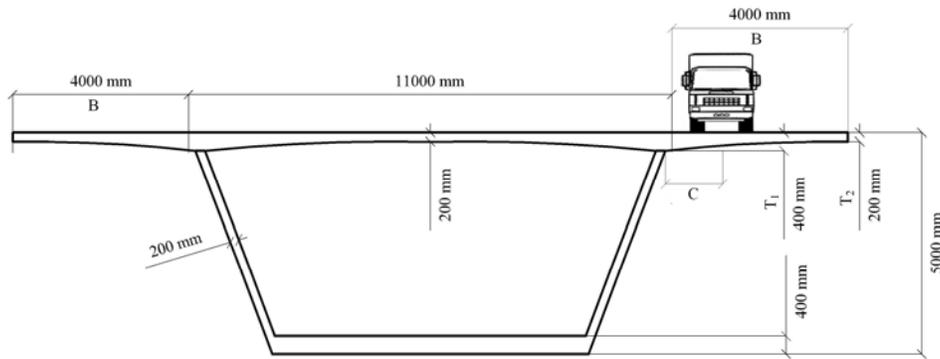


Figure 2. Conceptual cross-section of the reference model

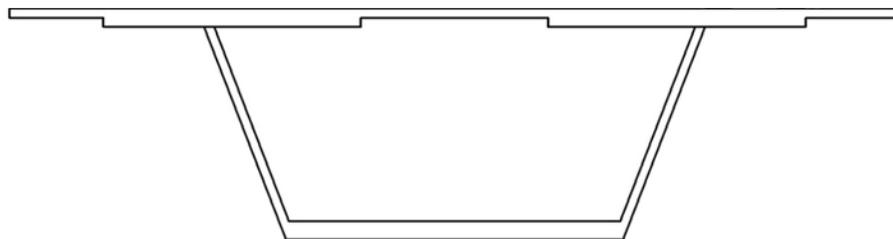


Figure 3. Simplified cross section of box girder bridge approximated for this study

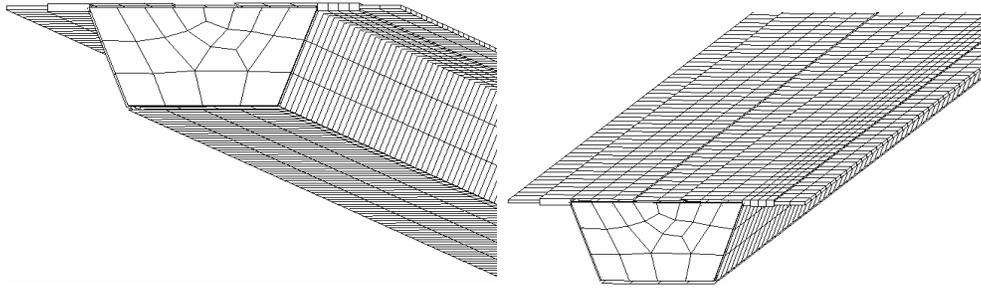


Figure 4a. Finite element model of the box girder bridge

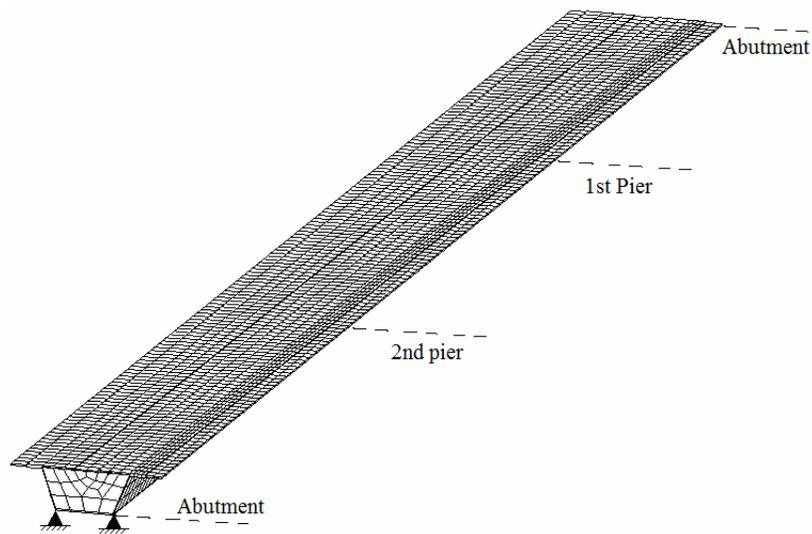


Figure 4b. Finite element model of the box girder bridge

2.2 Finite element modeling of cantilever bridge deck

Three dimensional modeling of the bridge has been done by using the finite element analysis software ANSYS 10.0. Four node shell element with six degrees of freedom at each node has been used to model the box girder bridge of concrete deck. Three dimensional orthographic views of the Finite Element Model are shown in Figure 4a and Figure 4b.

2.3 Vehicle Modeling

2.3.1 Vehicle loading simulation

The vehicle has been idealized as three concentrated forces moving along the deck in a path parallel to the centerline of the bridge at different axle position. The vehicle has been assumed to move at a constant velocity. Vehicle mass has been ignored.

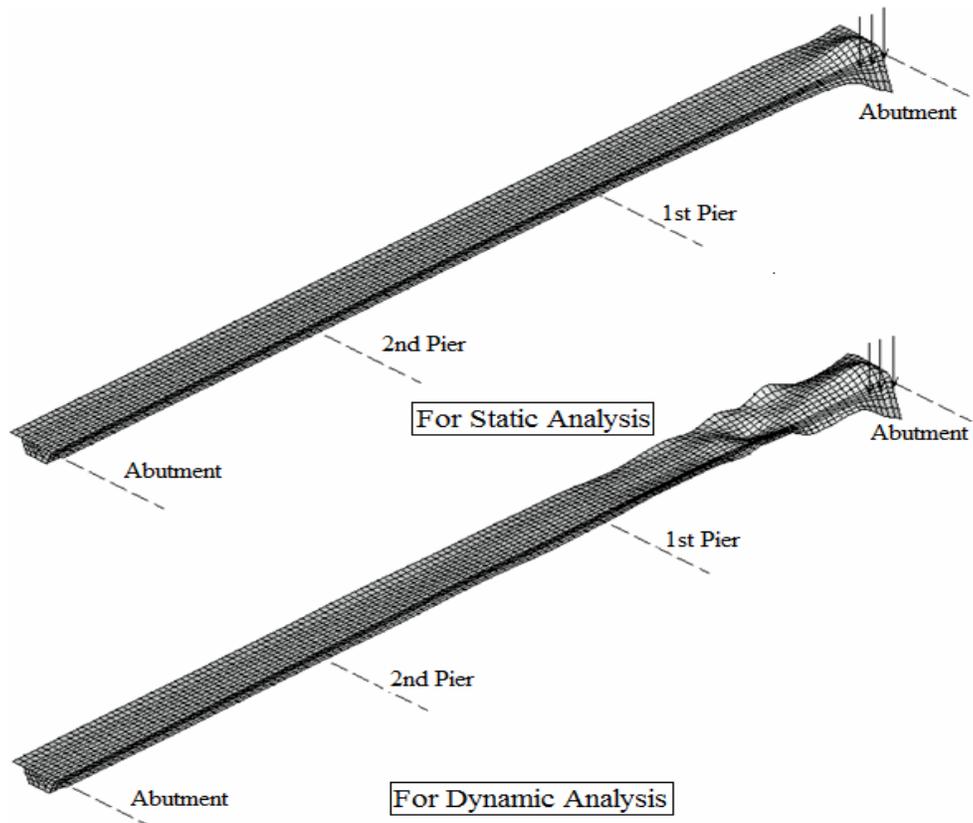


Figure 5. Animated deflection for $t=0$ sec at vehicle speed 100 km/h

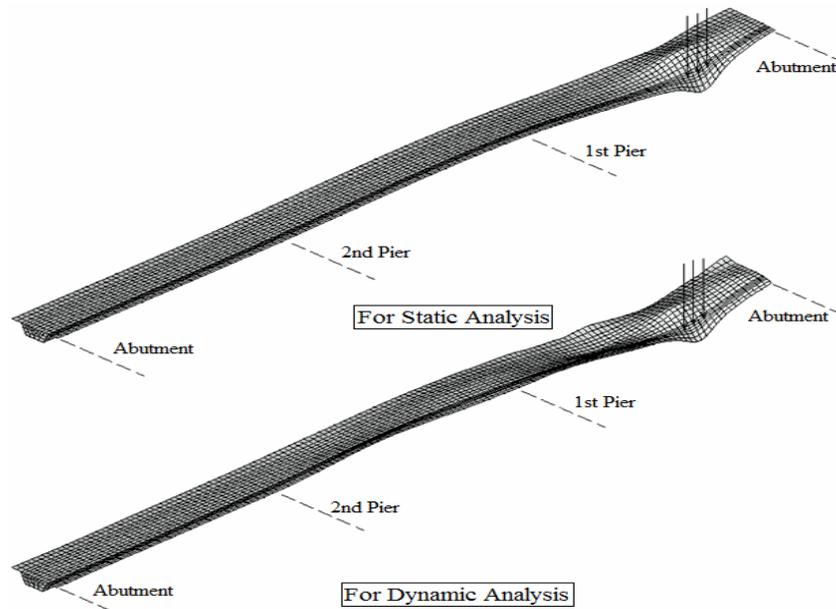


Figure 6. Animated deflection (for $t=1$ sec at vehicle speed 100 km/h)

$$\Delta t = \frac{\text{Dimension of the mesh along vehicle trajectory}}{\text{Vehiclespeed}}$$

2.3.2 Vehicle speed simulation

Vehicle speed has been simulated in such a way that forces coming from vehicle loads are applied on each consecutive node for a time period Δt ; where Δt is the time required for the vehicle to pass the division size along the longitudinal direction.

After each time period Δt , vehicle had to pass the next series of nodes consecutively. Stepped type loading has been considered on a particular node for a time period Δt instead of ramped load. The dynamic equilibrium equation at time $(t + \Delta t)$ has been solved by adopting Newmark integration formulae consecutively with the help of ANSYS software by default.

2.4 Total deflected shape of span for static and dynamic analysis

Animated deflections for static and full transient dynamic analyses are shown in Figure 5, Figure 6 and Figure 7 for 0 sec (vehicle at abutment), 1 sec and 4 sec (vehicle at 1st pier) respectively at a constant vehicle speed 100 km/h. Deformed shapes for vehicle at pier and midspan locations are shown in Figure 8.

2.5 Calculation of dynamic amplification factor

Deflections have been measured at various nodal points along the tip of the cantilever of the central span. The deflected shape is shown in Figure 9. The nodes i , j and k has been subjected to vertical displacement of magnitude U_1 , U_2 and U_3 respectively. The deformation U_3 consisted of two components viz. Rigid Body Movement U_4 and Deflection δ .

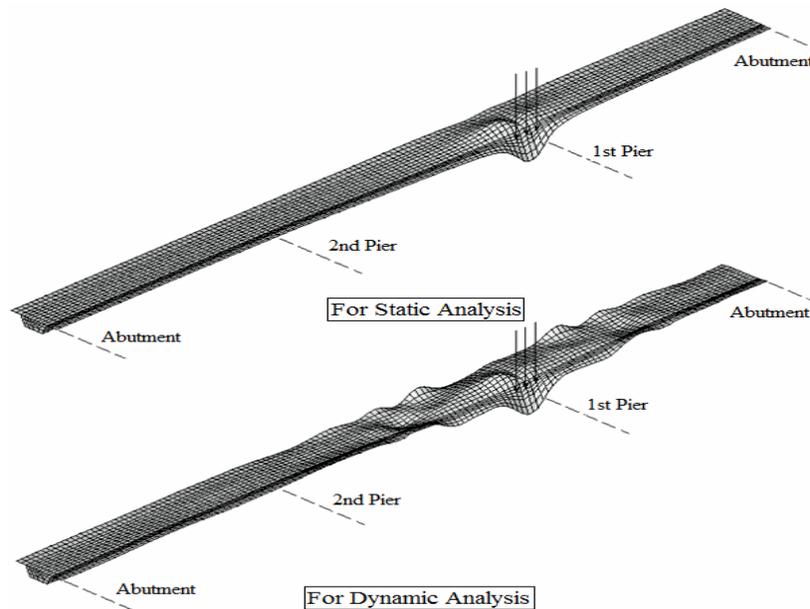


Figure 7. Animated deflection (for $t=4$ sec at vehicle speed 100 km/h)

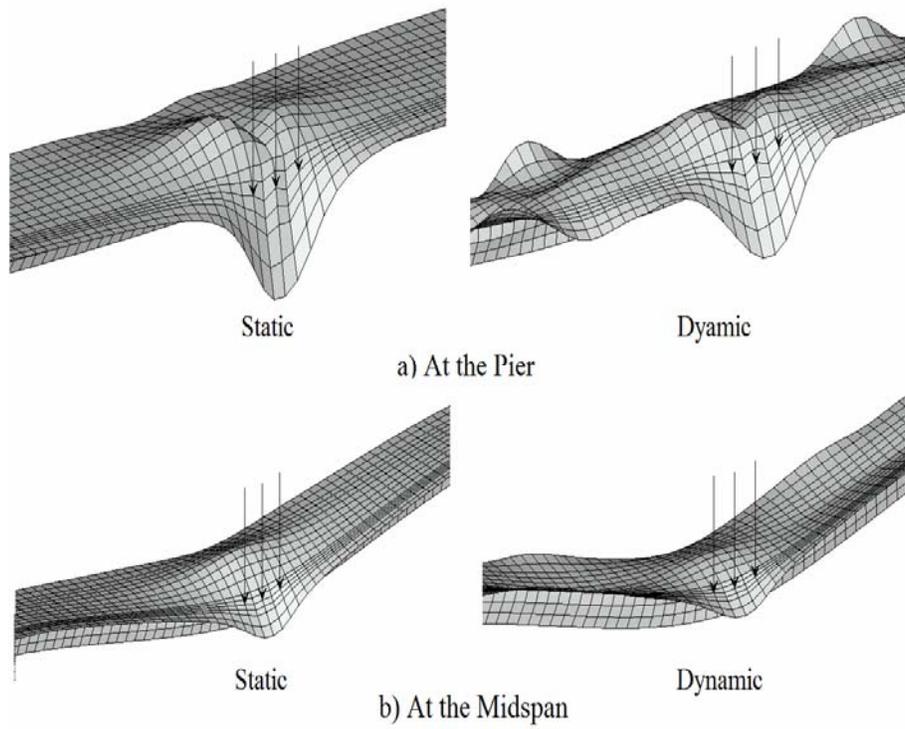


Figure 8. Deformed shape of the central span

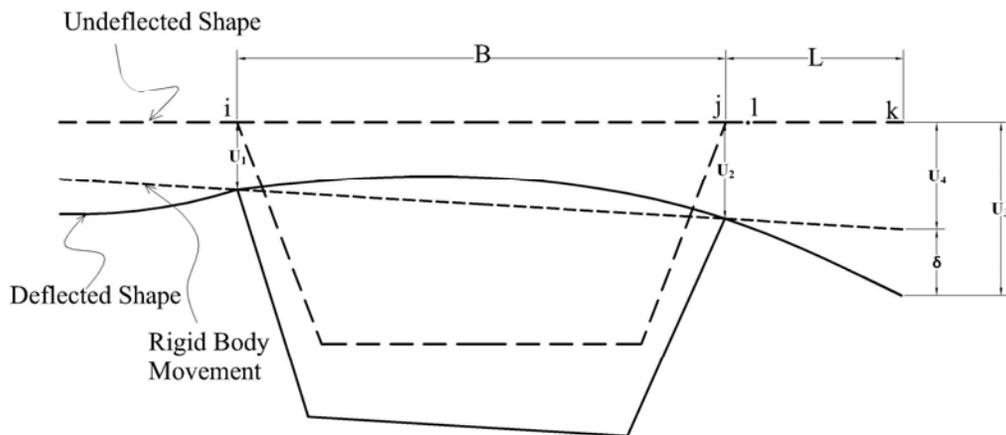


Figure 9. Deflected shape of the box girder (cross section)

Geometrically,

$$(U_2 - U_1) \frac{L+B}{B} + U_1 = U_4$$

$$(2.1)$$

$$\delta = U_3 - U_4$$

$$(2.2)$$

$$DAF_{Deflection} = \delta_{Dynamic} / \delta_{Static} \quad (2.3)$$

$$DAF_{Stress} = \sigma_{Dynamic} / \sigma_{Static} \quad (2.4)$$

For a particular vehicle speed, vehicle loads have been applied at a trajectory parallel to the edge of the cantilever and displacements have been measured for both static and full transient dynamic analyses at each of the specific nodes defined for displacement measurement. By using Equation (2.1) and (2.2), the magnitudes of deflections have been obtained at definite nodes at the tip of the cantilever. For each position of the vehicle, ratio of dynamic deflection to static deflection at a particular node has been measured. The absolute maximum ratio has been taken as the DAF for that particular node as shown in Equation (2.3).

Stresses have been measured at some definite nodes (node 1 as shown in Figure 9) a little far from the root of the cantilever. Stresses have been evaluated for both static and full transient dynamic analyses at each of these nodes for stress measurement. For each position of the vehicle, ratio of dynamic stress to static stress at a particular node has been measured. The absolute maximum ratio has taken as the DAF for that particular node as shown in Equation (2.4).

To avoid fictitious errors, lower values of deflections and stresses have been ignored for static loading. Only deflections and stresses having a magnitude greater than one-fourth of the maximum deflection and stress respectively at a node have been considered for static loading condition. Otherwise, the ratio of two insignificant small values might be too high and consequently the value of DAFs might be found too large.

3. Results

In this chapter, different influencing parameters are investigated numerically for both static and dynamic analyses in order to obtain the DAF and have a clear understanding of the effect of different parameters on the DAF based on the simulation of bridge vehicle interaction. These analyses have been performed to observe the characteristic properties of the dynamic responses for the cantilever part of the deck slab of concrete box girder bridges as well as the distribution of the DAFs in the transverse direction throughout the pertaining portion for deflection and stress measurement.

3.1 Numerical Analysis

Static analysis has been carried out to identify the effects in the cantilever part of the deck slab due to static traffic loading. Full transient dynamic analysis has been carried out to identify the effects in the cantilever part of the deck slab due to forced vibration. Vehicle wheel load has been applied as point loads at every node of its trajectory for a particular time. At each step the dynamic equilibrium between the vehicle and the bridge has been solved. The vertical deflection has been measured at the tip of the cantilever at different locations of the bridge. Figure 10a and Figure 10b illustrate the time history static responses and dynamic responses at the pier in terms of vertical deflections at the tip and transverse stresses at the root of the cantilever respectively. The peak values of deflection and stress for both static and dynamic analysis was found at 7.5 sec when the vehicle is at the proximity of the pier.

3.2 Parametric results

Parameters involved for the study of any kind of bridges are generally vehicle speed, span length, and slab width. In this paper, Investigations have been done for span length only, while the other parameters were kept constant. Thus, detailed parametric study has been carried out for determining dynamic amplification factor as a function of span length.

The simulations have been carried out for 7 different span lengths with the vehicle passing along the line located at a distance 1 m from the root of the cantilever at a speed of 100 km/h. Different span lengths have been coupled with simultaneous changes in slab thicknesses.

3.2.1 Variation along the length of the central span in terms of vertical deflections

The variations of the DAF along the central span are shown in Figure 11a through Figure 11g in terms of vertical deflections at the tip of the cantilever for various span lengths. Figure 11a illustrates the variation of the DAF along the length of the central span for 25 m span. The range has been found to vary between 0.51 and 2.21. The minimum DAF has been found to appear at the first pier; whereas the maximum DAF occurred at a distance $5L/6$ and $11L/12$ from the first pier. The DAFs have been found to remain more or less constant throughout most of the central span.

For span lengths 50 m to 175 m at intervals of 25 m, variation of the DAF for the central span are shown in Figures 11b through Figure 11g. In all of these figures, the variation of the DAF is fundamentally of similar nature to that of Figure 11a. Though maximum magnitude for DAF is different for different span lengths, apparently no direct correlation between maximum DAF and span length could be observed. For example, maximum value of DAF of about 3.01 occurs for 100 m span at the second pier.

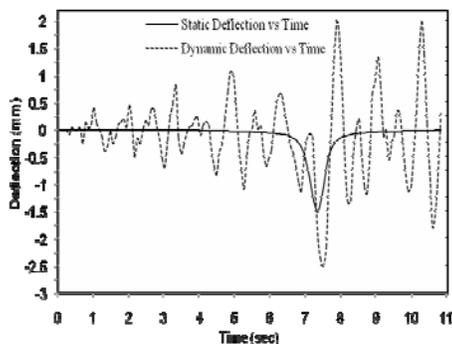


Figure 10a: Static and Dynamic Responses of Vertical Deflection at the Root of the Cantilever at Vehicle Speed 100 km/h for the Central Span

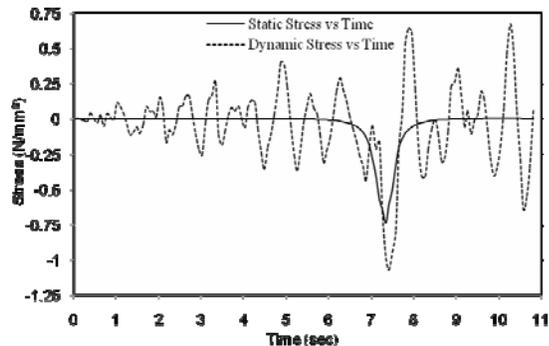
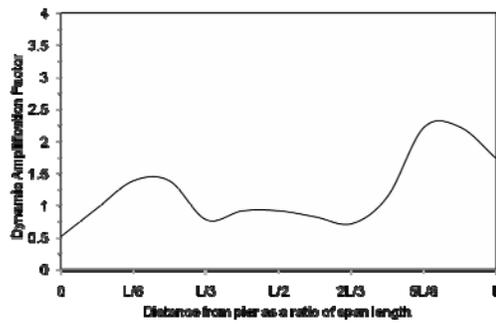
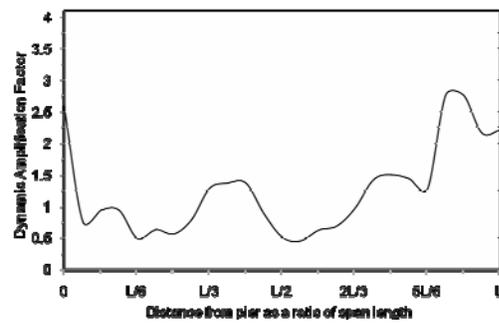


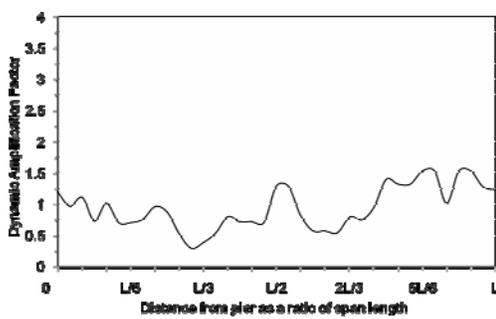
Figure 10b: Static and Dynamic Responses of Transverse Stresses at the Root of the Cantilever at Vehicle Speed 100 km/h for the Central Span



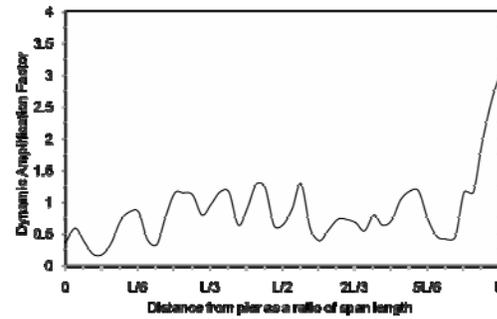
a) For 25 m Span Length



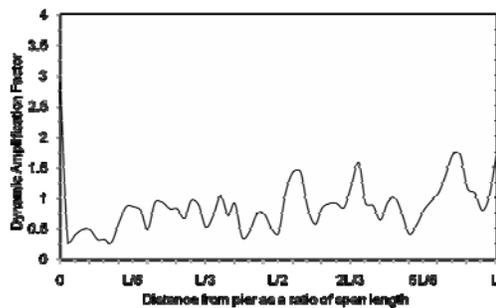
b) For 50 m Span Length



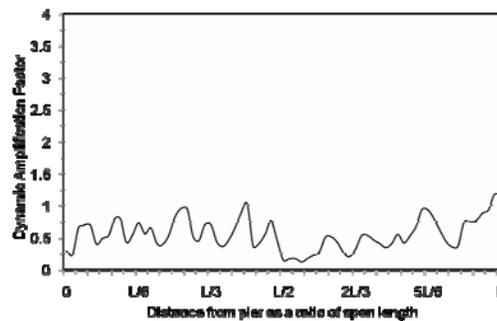
c) For 75 m Span Length



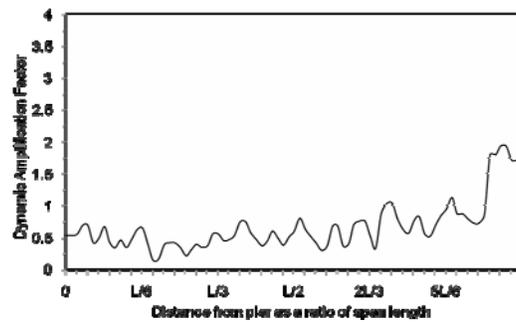
d) For 100 m Span Length



e) For 125 m Span Length

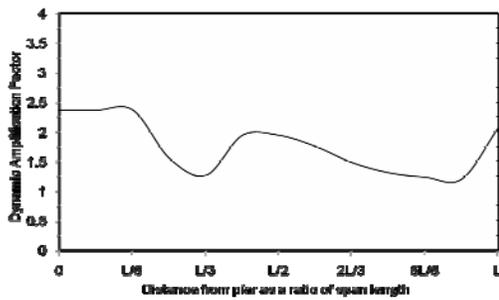


f) For 150 m Span Length

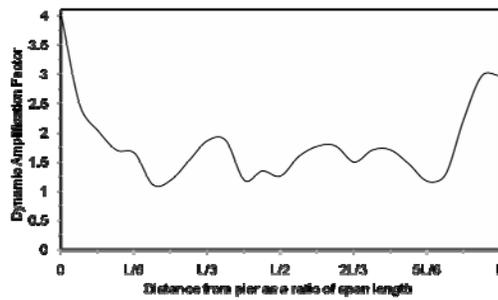


g) For 175 m Span Length

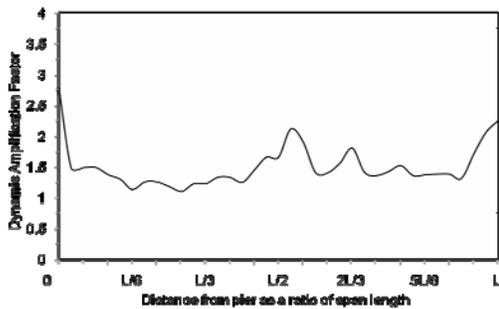
Figure 11. Variation of Dynamic Amplification Factor in terms of Vertical Deflections along the Length of the Central Span at 100 km/h vehicle speed for 7 Different Span Length



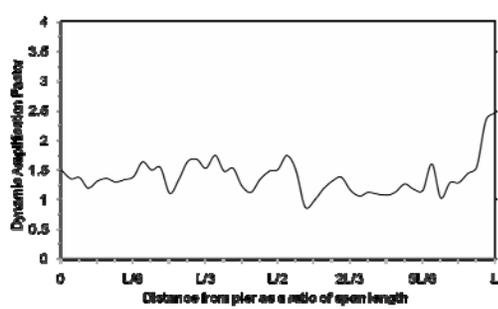
a) For 25 m Span Length



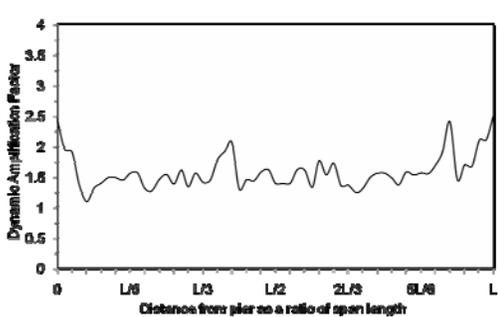
b) For 50 m Span Length



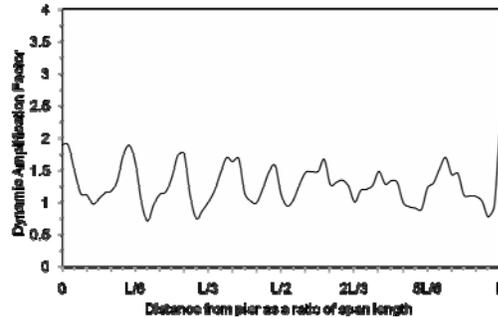
c) For 75 m Span Length



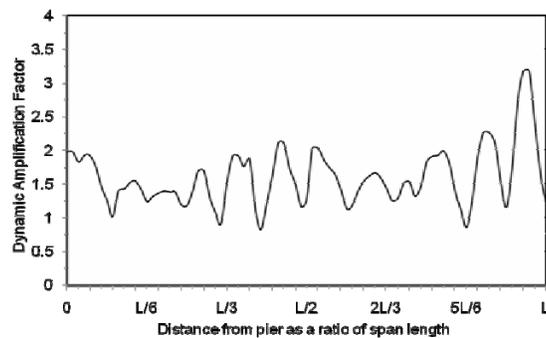
d) For 100 m Span Length



e) For 125 m Span Length



f) For 150 m Span Length



g) For 175 m Span Length

Figure 12. Variation of Dynamic Amplification Factor in terms of Transverse Stresses along the Length of the Central Span at 100 km/h vehicle speed for 7 Different Span Length

In most cases, the maximum DAFs are found to occur at the vicinity of both of the piers of the central span. The DAF was found to vary from 0.141 to 3.01 along the central span for various span lengths. Trivial values of 0.5 to 1.0 are common for a major portion of the central span, whereas extreme values of 2.0 to 3.0 could be observed at the proximity of both of the piers.

3.2.2 Variation along the length of the central span in terms of transverse stresses

The variations of the DAF along the central span are shown in Figure 12a through Figure 12g in terms of transverse stresses at the root of the cantilever for various span lengths. Figure 12a illustrates the variation of the DAF along the length of the central span for 25 m span. The range has been found to vary between 1.20 and 2.37. The minimum DAF has been found to appear at a distance $11L/12$ from the first pier; whereas the maximum DAF occurred at a distance $L/12$ and $L/6$ from the first pier. The DAFs were found to remain more or less constant throughout most of the central span.

For span lengths 50 m to 175 m at intervals of 25 m, variation of the DAF for the central span are shown in Figures 11b through Figure 11g. In all of these figures, similar kind of variations of the DAF has been observed. No direct correlation between maximum DAF and span length could be noticed, as maximum magnitude for DAF is found to vary arbitrarily with the change in span length. For example, maximum value of DAF of about 4.02 occurs for 50 m span at the first pier.

In most cases, the maximum DAFs are found to occur at or near the two piers of the central span. DAF was found to fluctuate over a wide range of 0.70 to 4.02 along the central span. Most of the obtained DAF values were above unity. A considerably larger range of values varying from 1.0 to 2.0 is obtained for a major portion of the central span. Extreme values of 2.0 to 4.0 could be observed at the proximity of both of the piers.

On both occasions, higher values of DAFs were observed on nodes that are at or near the piers of the central span. It is because when the axles of the vehicle enter the central span from the adjacent span, it causes massive dynamic impact on the deck and consequently results in higher values of DAFs. Similar incident happens while the axles of the vehicle move from the central span to the next span causing sudden release of the vehicular impact force which contributes to higher values of DAFs.

3.2.3 Variation DAFs with respect to span length at the piers and midspan

The variation of the DAF in terms of deflections and stresses are shown in Figure 13 and Figure 14 respectively as a function of span length. The results are shown at the three critical locations viz. first pier, midspan, and second pier for a constant vehicle speed of 100 km/h.

In Figure 13, DAF has been found to vary from 0.28~3.01 at the piers and midspan. The maximum and minimum DAFs have been found for different span lengths at different locations.

From this graph, no clear relationship could be developed between the DAF and span length which concludes that the DAF in terms of deflections vary arbitrarily with the change in span length.

In Figure 14, the DAF has been found to vary from 1.2 to 4.02 at the piers and midspan. An excessively high range of values have been obtained, which shows that the DAFs at

these locations are crucial. The maximum and minimum DAFs were found for different span lengths at different locations.

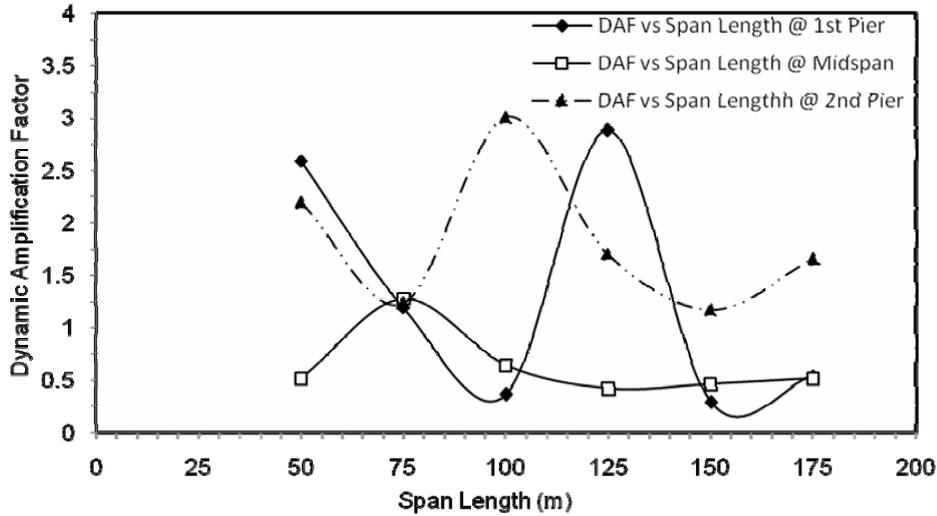


Figure 13. Dynamic Amplification Factor in terms of Deflections as a Function of Span Length at Vehicle Speed 100 km/h

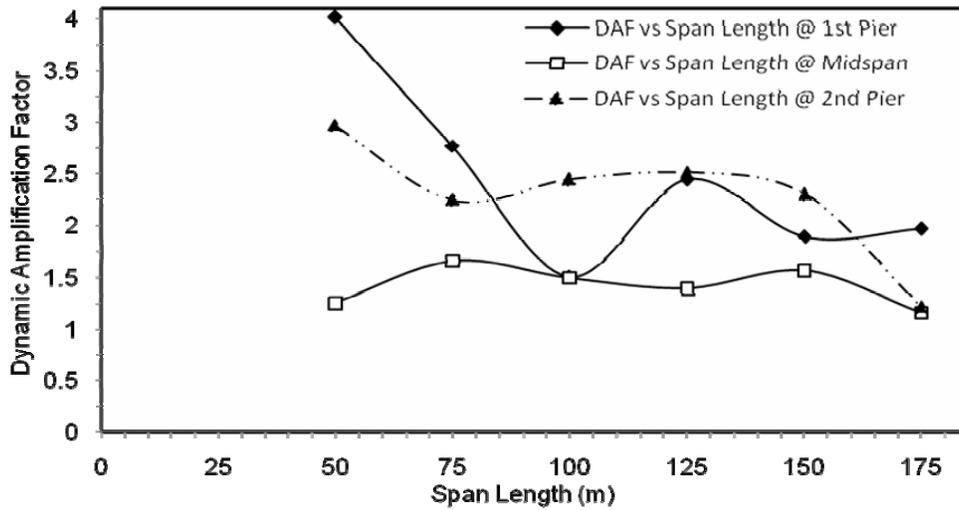


Figure 14. Dynamic Amplification Factor in terms of Stresses as a Function of Span Length at Vehicle Speed 100 km/h

Though no direct relationship could be developed from this graph, it can be observed that the DAF decreases with the increase in span length. It has also been noted that stresses at the pier are critical for span length variation analysis.

4. Conclusions

The major objective of this study has been to expand our knowledge about the influence of span length on the amplitude and distribution of peak static and transient responses of the cantilever part of the box girder bridge due to moving vehicular load. The objective has also been to recommend a feasible value of Dynamic Amplification Factor in terms of vertical deflections at the tip of the cantilever and transverse stresses at the root of the cantilever.

On the basis of numerical analysis, it has been found that the DAF varies along the span in an irregular manner. For various span lengths, high values of DAF have been obtained at the proximity of both of the piers of the central span. The pier region has been found more vulnerable to dynamic load than the central part of the span as the DAFs at the intermediate points along the span have been found somewhat smaller. The DAF has been found not to vary proportionately with the span length but rather in an arbitrary or irregular manner as a result no direct correlation could be developed between the Dynamic Amplification Factor and span length.

The overall findings offer to take a standard Dynamic Amplification Factor of 3.0 for deflections of the cantilever part of deck slab i.e. recommending 200% impact increment over static results calculated in terms of deflections. It can be observed that Dynamic Amplification Factor is more critical when measured in terms of stresses than that in terms of deflections, as higher values have been obtained for stress analysis. Studies suggest taking a standard DAF of 4.0 for stresses in the transverse direction of the cantilever part of deck slab i.e. recommending 300% impact increment over static results calculated in terms of transverse stresses.

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