

# Modified Bolomey equation for the design of concrete

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## **Abstract**

Bolomey equation and modified Bolomey equations are used for the design of light weight concrete. In India huge quantities of soap stone wastes are produced from stone carving industries every year. Soap stone is regarded as unconventional aggregate for the production of concrete. The soap stone aggregates are weaker and denser with lower values of water absorption. In spite of this soap stone has lots of potential for use in concrete production. This paper deals with development of an appropriate technology to know the aggregate characteristic strength in concrete and proportion mortar strengths to higher concrete strength or to limit the strength of concrete to that of the aggregate strength for optimal use of cementing materials. For this well established generalized Abrams law based on composite mechanics approach and modified Bolomey equation are successfully used. Few concrete mixes are designed containing soap stone as well as granite stone as coarse aggregates. Based on the results, correction to the published modified Bolomey equation is also suggested which can be used for the design concrete containing soap stone.

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*Keywords:* Industrial wastes, Re-proportioning concrete mixtures, Bolomey equation, Compressive strength of concrete

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## **1. Introduction**

Depletion of natural resources is a common phenomenon in developing countries like India due to rapid urbanization and industrialization. Due to this engineers are in search of suitable alternative materials for concrete so that the existing natural resources could be preserved for the future. One such example is the industrial wastes such as fly ash, blast furnace slag, quarry dust, tile waste, brick bats, broken glass waste, demolition wastes, etc. One promising material in this direction is the soap stone produced from carving industries. Soap stone can be used either as a partial or full replacement to the conventional coarse aggregate in concrete. Research information regarding studies on concrete made with different wastes is available in different forms in a scattered manner around the world. However a well established method for the design of concrete containing unconventional aggregates is not

available. Only a few cases have been reported on the use of recycled aggregates in structural concrete, and the amount of recycled aggregate used has generally been limited to a low level of replacement [1-4]. Use of recycled aggregate in India is very limited and is picking up slowly. The limited use of recycled aggregate in structural concrete is due to certain inherent deficiencies of this type of material. In comparison with natural normal weight aggregates, recycled aggregates are weaker, more porous and have higher values of water absorption [5]. In contrast soap stone is less porous, denser and soft in nature. This paper presents a recent study by the authors which aims to develop a technique for using 100% of recycled soap stone aggregates in concrete. This aggregate is obtained from broken soap stone cuttings generated in a local industry. The waste is crushed to 20 mm or less for use in concrete as coarse aggregate. Soapstone consists mainly of talc, which contain silicon oxide, oxygen, magnesium oxide, water and loose molecules of magnesium and silicon. On the hardness scale that places diamond at 10, soapstone is at 3. Soapstone varies in color depending on other incidental minerals mixed into its creation.

## 2. Experimental investigation

### 2.1 Materials used

In the present experimental study, the cement, coarse aggregates, fine aggregate, water and superplasticizer were used.

#### Cement

43 Grade OPC conforming to IS: 8112 [6] was used. The results obtained from the tests are shown in Table 1.

Table 1  
Properties of 43 grade OPC

Sl. No.	Properties	Test Results	IS: 8112-1989 Requirements
1	Standard Consistency, %	30.25	No standard value
2		Setting time in minutes	
	Initial setting time	135	Not less than 30
	Final setting time	225	Not greater than 600
3	Specific gravity	3.15	-
4		Compressive Strength (MPa)	
	3 days	28.13	23
	7 days	40.18	33
	28 days	56.17	43

Table 2  
Properties of soap stone and granite aggregates as obtained in laboratory

Properties	Soap stone aggregates	Granite aggregate
Specific gravity	2.85	2.61
Dry rodded density (oven dry)	1665 kg/m <sup>3</sup>	1634
Dry rodded density (ssd)	1731 kg/m <sup>3</sup>	1639
Water absorption	1.04 %	0.41
% void	41.75	35.21
Impact value	9.06	22.82
Crushing value	9.93	23.45
Hardness	> 2	> 5

*Coarse aggregates*

Crushed soap stone obtained from local soap stone industry and locally available broken granite is used as coarse aggregates. The aggregates passing through 20 mm and retained on 4.75 mm are used with 60:40 proportion which satisfies the requirements as per IS: 383 [7]. The composition of soap stone and properties of both aggregates are presented in Table 2. The aggregates are tested as per IS:2386 [8].

*Fine aggregate*

Naturally available river sand conforming to zone II as per IS: 383 have been used as fine aggregate. The specific gravity, fineness modulus and water absorption of fine aggregate are 2.61, 2.85 and 0.48 %, respectively.

*Water*

Tap water is used for mixing and curing of concrete and mortar cubes.

*Superplasticiser*

In the present investigation super plasticizing admixture is used, which complies with IS: 9103:1979 [9]. Conplast SP 430 is based on sulphonated naphthalene polymers and is supplied as a brown liquid instantly dispersible in water. It has been specially formulated to give high water reduction upto 25% without loss of workability. Its specific gravity is 1.145 (at 30°C) and chloride content is Nil. Air entrainment is approximately 1%.

### 3. Proportioning and re-proportioning of concrete mixes

In the present investigation for the trial mix proportioning ACI 211.2 method [10] is used.

Steps involved in arriving at the trial mix proportions are

- a) Volume of concrete = 1 m<sup>3</sup>
- b) Volume of cement = weight of cement/[specific gravity of cement × 10<sup>3</sup>]
- c) Volume of entrapped air
- d) Volume of water = weight of water/[specific gravity of water × 10<sup>3</sup>]
- e) Volume of admixture = [weight of admixture/ [specific gravity of admixture× 10<sup>3</sup>]
- f) Volume of coarse aggregate = weight of coarse aggregate/[specific gravity × 10<sup>3</sup>]
- g) Volume of fine aggregate=1- (b + c + d + e +f)
- h) Weight of fine aggregate = specific gravity × volume fraction × 10<sup>3</sup>

In concrete technology, the concrete mix proportioning mainly depends on the Abrams law according to which it has been categorically stated that, *for a given set of materials*, the strength development is solely dependent on free water–cement ratio. In other words as cement or combinations of cementitious materials and/or aggregate characteristics such as size, shape and surface characteristics change, even if the water – cement ratio is the same the strength development is not the same. Owing to this, a trial mix is arrived at based on empirical considerations and tested for its strength. The strength obtained for this trial mix might not meet the practical requirements. Hence an adjustment to water–cement ratio has to be made until it is possible to arrive at the water–cement ratio required to arrive at the final mixture proportions so as to meet the practical strength requirements envisaged [11-12].

In concrete, mortar is regarded as the matrix and coarse aggregate is the distributed phase. It is found that soap stone aggregate is not as strong as conventional natural aggregate even though its specific gravity is higher than that of granite. According to the law of mixture of



## 5. Trial mix results

Properties of materials and trial mix details for concrete containing soap stone aggregates, granite aggregates and corresponding mortar cubes for a water cement ratio of 0.5 (reference) are presented in Tables 3 and 4 respectively. The workability and the compressive strength of concrete at 7 and 28 days are shown in Table 5.

Table 4  
Trial mix details per cubic meter concrete

Ingredients	Crushed soap stone as coarse aggregate	Crushed Granite as coarse aggregate
	Concrete	
Water Cement ratio [W/C]	0.5	0.5
Water content [kg/m <sup>3</sup> ]	202	202
Cement content [kg/m <sup>3</sup> ]	404	404
Fine Aggregate [kg/m <sup>3</sup> ]	668	668
Coarse Aggregate [kg/m <sup>3</sup> ]	1176	1074
Aggregate Cement ratio [A/C]	4.56	4.30
Superplasticizer [%]	0%	0%
	Mortar mix	
Water Cement ratio [W/C]	0.5	0.5
Fine Aggregate Cement ratio [FA/C]	1.65	1.65

Table 5  
Compressive strength and workability of trial mix concrete and mortar

Type of Mix	Compressive Strength, MPa		Workability	
	7 days	28 days	Slump, mm	Compaction factor
Crushed Granite as coarse aggregate (SSD)	33.27	44.47	70	0.94
Crushed soap stone as coarse aggregate (SSD)	25.34	36	50	0.93
Equivalent Mortar*	32.02	50.15	160 (Collapse)	1 (Flow)

\*Mortar proportion is the same for both aggregates (1:1.65)

From the test results of trial mix, the characteristic strength of both aggregates is determined as explained in the next section.

## 6. Determination of aggregate characteristic strength

The term aggregate characteristic strength is the strength contributed from the coarse aggregate. With the concrete and mortar strengths and their respective volume fractions as input parameters, the aggregate characteristic strength is determined from linear law of mixtures. This law as an equation is from composite mechanics consideration and is given by;

$$\sigma_c = \sigma_m V_m + \sigma_a V_a$$

where,

$\sigma_c$  = Strength of concrete

$\sigma_m$  = Strength of constituent mortar

$\sigma_a$  = Characteristic strength of aggregate

$V_a, V_m$  = Volume fraction of aggregate and mortar

$V_a$  = Weight / Specific gravity

$V_m = 1 - V_a$

For granite aggregate, we have

$$V_a = \text{Weight/ Specific gravity} \\ = 1074 / (2.61 * 1000) = 0.412.$$

$$V_m = 1 - 0.412 = 0.588.$$

$$\sigma_a = [\sigma_c - \sigma_m V_m] / V_a \\ = [44.47 - (50.15 * 0.588)] / 0.412 = 36.36 \text{ MPa.}$$

Similarly for soap stone we have

$$V_a = \text{Weight/ Specific gravity} \\ = 1176 / (2.85 * 1000) = 0.412.$$

$$V_m = 1 - 0.412 = 0.588.$$

$$\sigma_a = [\sigma_c - \sigma_m V_m] / V_a \\ = [36 - (50.15 * 0.588)] / 0.412 = 15.80 \text{ MPa.}$$

Using these characteristic strengths of aggregates found from the 28day strength, the 7 day strength of concrete is predicted as follows and it matches with the experimental values.

*For granite aggregate concrete*

$$\sigma_c = \sigma_m V_m + \sigma_a V_a$$

$$\sigma_c = (32 \times 0.588) + (36.36 \times 0.412) = 33.79 \text{ MPa as against } 33.27 \text{ MPa.}$$

*For soap stone aggregate concrete*

$$\sigma_c = \sigma_m V_m + \sigma_a V_a$$

$$\sigma_c = (32 \times 0.588) + (15.80 \times 0.412) = 25.33 \text{ MPa as against } 25.34 \text{ MPa.}$$

## 7. Re-proportioning based on aggregate characteristic strength

It was noticed that the compressive strength of the constituent mortar is higher compared to the corresponding concrete. The mortar matrix strength is also greater than the aggregate strength and hence iso-strain condition prevails. The concrete cubes were failed predominantly by crushing of the aggregate. The fracture was through the aggregate. Hence by considering this failure, it is suitable to analyze the strength by law of mixtures. This value of  $\sigma_a$  (15.80) is used to re-proportion the concrete containing soap stone as the fracture is essentially through the aggregate. Here re-proportioning of mixes is done for target strengths of 25, 30, 35, 45 and 50 MPa using soap stone aggregates and for target strength of 30 and 60 MPa for granite aggregates. These mixes are called M25S, M30S, M35S, M45S and M50S for soap stone and M30G and M60G for granite respectively. One typical mix design for M25S is illustrated here. The w/c ratio is found using the method of re-proportioning. The mix details for all mixes consisting of soap stone and granite are presented in Table 6.

We know that,  $S/S_{0.5} = -0.2 + 0.6 * (C/W)$  for  $S_{0.5} > 30$  MPa

Aggregate strength < required concrete strength

$$\sigma_c = \sigma_m V_m + \sigma_a V_a$$

$$\sigma_m = [\sigma_c - \sigma_a V_a] / V_m$$

$$\sigma_m = [25 - (15.80 * 0.412)] / 0.588$$

$$\sigma_m = 31.44 \text{ MPa}$$

$$31.44 / 50.15 = -0.2 + 0.6 * (C/W)$$

Solving,  $w/c = 0.75$

Water for 20 mm downsize aggregate and a slump of 75-100mm is 202 kg. Hence, Cement =  $202/0.75 = 269.33 \text{ Kg/m}^3$

Coarse aggregate = Volume fraction x dry rodded density x correction for water absorption  
 $= 0.70 \times 1665 \times 1.0104 = 1176 \text{ kg/m}^3$

Keeping the CA content same and for the above  $w/c$ , calculate re-proportioned constituents of the mix using ACI 211.2 as follows;

- |                                 |  |
|---------------------------------|--|
| a) Volume of concrete           | $= 1 \text{ m}^3$  |
| b) Volume of cement             | $= 269.33 / 3.15 \times 10^3 = 0.085$                    |
| c) Volume of entrapped air (2%) | $= 0.02$   |
| d) Volume of water              | $= 0.202$  |
| f) Volume of aggregate          | $= 1176 / 2.85 \times 10^3 = 0.412$                      |
| g) Volume of fine aggregate     | $= 1 - (0.085 + 0.02 + 0.202 + 0.412) = 0.281$           |
| h) Weight of fine aggregate     | $= 2.60 \times 0.281 \times 10^3 = 730.6 \text{ kg/m}^3$ |

Giving, Cement: Fine Aggregate: Coarse Aggregate to be 1:2.7:4.37

Table 6  
 Mix details for soap stone and granite aggregate concrete

Type of Mix	Cement, kg	Fine aggregate, kg	Coarse aggregate, kg
M25S, $w/c = 0.75$ , $w = 202 \text{ kg}$	269	731	1176
M30S, $w/c = 0.60$ , $w = 202 \text{ kg}$	337	676	1176
M35S, $w/c = 0.51$ , $w = 202 \text{ kg}$	396	627	1176
M45S, $w/c = 0.40$ , $w = 202 \text{ kg}$	505	588	1176
M50S, $w/c = 0.35$ , $w = 202 \text{ kg}$	577	470	1176
M30G, $w/c = 0.68$ , $w = 202 \text{ kg}$	297	759	1074
M60G, $w/c = 0.34$ , $w = 202 \text{ kg}$	594	514	1074

## 8. Concrete testing

Concrete cubes for all the above mixes are cast and tested as per IS: 516 -1959 [16] and the results are presented in Table 7. All mixes are tested for workability in terms of slump and compacting factor (CF) as per the Indian Standard IS: 1199-1959 [17]. From Table 7, it is clear that the 28 days strength obtained is almost equal to the required design strength for both concrete indicating the applicability of generalized Abrams law.

Table 7  
Compressive strength and workability of re-proportioned mixes

Type of Mix	Compressive Strength $f_c$ from lab MPa		Workability	
	7 days	28 days	Slump, mm	Compaction factor
M25S*	14.23	24.34	50	0.95
M30S	20.44	30.64	75	0.94
M35S	23.67	34.53	60	0.94
M45S	30.89	44.23	50	0.91
M50S	40.33	49.06	40	0.88
M30G	22.81	32.22	65	0.92
M60G	41.23	57.02	20	0.85

\*  $[S/S_{0.5} = -0.2 + 0.6 (C/W)]$ , for  $S_{0.5} \geq 30$  MPa

### 9. Use of the Modified Bolomey equation for the design of light weight aggregate concrete

The Modified Bolomey equation is suggested by Rajamane and Ambily [18] for the design of light weight concrete containing fly ash aggregates. This equation is generalized for w/c of 0.4. In order to check the applicability of this method to our experimental results, their published work has been studied in detail and the equation is further modified taking the generalized w/c ratio as 0.5. This is done as generalized Abrams law is for w/c of 0.5 such that all results can be compared. It is well established that generalized Abrams law is applicable to all types of concrete and for any type of aggregates. The modified Bolomey equation proposed by Rajamane and Ambily and the further modified Bolomey equation proposed by the authors in the present work are of the form,

$$f_c = f_{0.4}[0.50 c/w - 0.25]^{(1-a)}, \text{ [Later called MBE(0.4)]} \quad (3)$$

$$f_c = f_{0.5}[0.67 c/w - 0.34]^{(1-a)}, \text{ [later called MBE(0.5)]} \quad (4)$$

where,

$f_{0.4}$  is the compressive strength at w/c = 0.4

$f_{0.5}$  is the compressive strength at w/c = 0.5

c/w is cement to water ratio = 1/ (w/c)

a is the volume fraction of coarse aggregate

#### Observation

Using the above two equations, the published results of Rajamane and Ambily [18] is verified and presented in Table 8. Not much difference in the target and the observed strengths are noticed and the difference is within about 5% for most of the cases except few. From Table 8, it is observed that the strength predicted from MBE (0.4) is more by about 4 to 8%. It means actual strength obtained in the laboratory is slightly less. However MBE (0.5) predicts strengths slightly less by about 4% which is slightly less compared to MBE (0.4). It means the actual value obtained in the laboratory is more by about 4% for few mixes. Thus equation MBE (0.5) is more reliable and the percentage error is also less and conservative.

Table 8  
Applicability of the MBE for light weight concrete for published data [18]

Sl. No.	w/c	Density kg/m <sup>3</sup>	V <sub>f</sub>	f <sub>c</sub> (obtained in lab.)	Bolomey-MBE (0.4)	% difference MBE (0.4)	Bolomey-MBE(0.5)	% difference MBE(0.5)
1	0.40	2170	0.37	23.00	23.00	0.00	21.66	-6.18
	0.50	2170		18.00	19.20	6.10	18.00	0.00
	0.60	2170		16.00	16.30	2.00	15.43	-3.69
2	0.40	2173	0.36	20.00	20.00	0.00	21.69	7.78
	0.50	2141		17.00	16.70	-2.10	17.00	0.00
	0.60	2123		14.00	14.10	0.90	14.57	3.90
3	0.30	2.175	0.18	41.70	43.40	3.90	50.51	17.44
	0.40	2117		32.60	32.60	0.00	38.00	14.00
	0.50	2072		29.80	25.70	-15.70	29.80	0.00
4	0.30	2117	0.24	37.50	38.40	2.50	41.91	10.50
	0.40	2059		29.50	29.50	0.00	32.19	8.00
	0.50	2014		25.70	23.70	-8.40	25.70	0.00
5	0.30	2060	0.30	35.00	35.20	0.60	36.56	4.00
	0.40	2002		27.60	27.60	0.00	28.67	3.00
	0.50	1957		23.30	22.60	-3.30	23.30	0.00
6	0.30	2002	0.36	31.80	28.80	-10.60	31.15	-2.00
	0.40	1944		23.00	23.00	0.00	25.75	10.67
	0.50	1899		21.30	19.10	-11.40	21.30	0.00

The above two equations are further applied to our experimental data on soap stone aggregate concrete and the results are presented in Table 9. The results obtained from the generalized equation based on generalized Abrams law using composite mechanics approach is more close to MBE (0.5). Although all the three approaches give more or less the same predicted strength, the one obtained from generalized Abrams law is more close to the required strength. In addition generalized Abrams law requires only water-cement ratio and is independent on aggregate volume fraction. However in MBE, both water-cement ratio and the volume fraction are considered. However it is observed that volume fraction is not significant for the range of w/c ratio from 0.30-0.5. And it is also seen that as the volume fraction of light weight aggregate increases the compressive strength decreases marginally, and is found to be insignificant.

Table 9  
Application of modified Bolomey equation

Type of mix**	f <sub>c</sub> (Experiment)	f <sub>c</sub> from MBE(0.4)	% Difference	f <sub>c</sub> from MBE(0.5)	% difference
M25S, w/c =0.75, w=202kg	24.34	26.43	7.91	25.42	0.42
M30S, w/c =0.60, w=202kg	30.64	30.50	-0.45	29.35	-4.39
M35S, w/c =0.51, w=202kg	34.54	36.87	6.32	35.44	0.29
w/c =0.50	36.00	37.44	3.85	36.00	0.00
M45S, w/c =0.40, w=202kg	44.23	44.23	0	42.67	-3.66
M50S, w/c =0.35, w=202kg	49.06	48.64	-0.86	47.00	-4.38

\*\*Volume fraction for these mixes, v<sub>f</sub> = 0.412

## 10. Conclusions

Following conclusion can be drawn based on the observations and discussions

1. Aggregates obtained from soap stone are relatively heavy, rough and irregular compared to conventional granite aggregates. Their surface is rough when broken and slightly irregular in shape and their characteristic compressive strength is rather less. In spite of this, relatively good concrete can be produced from soap stone aggregates.
2. Here the concrete is designed for the required strength. For few mixes, the workability has decreased when soap stone aggregates were used which is mainly due to its surface characteristics. To compensate this, suitable dosage of superplasticiser should be added.
3. Concrete of medium strength in the range of 20 MPa to 40 MPa can be easily produced using soap stone aggregates.
4. Use of generalized Abrams law for the design of concrete containing soap stone is demonstrated successfully.
5. Technical feasibility of compensating the low characteristic strength of soapstone as coarse aggregate by commensurate mortar strength is demonstrated.
6. It is noticed that both the modified Bolomey equations and generalized Abrams law predicts the strength of concrete very well within about 5% errors.
7. Generalized Abrams law requires only water cement ratio where as Modified Bolomey equation requires both the water cement ratio and the volume fraction of coarse aggregate for design.

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