Journal of Civil Engineering (IEB), 50 (1) (2022) 1-11

Journal of
Civil Engineering
IEB

A simplified parametric analysis of laterally loaded single piles in homogenous soil

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Received 23 January 2022

Abstract

This paper presents a simplified method in determining the maximum moment and maximum deflection of a free headed pile in homogenous soil under the effect of lateral load. Various soil types (loose, medium, dense) proposed by Terzaghi are utilized here. Both the normalized deflection curve and normalized moment curve are continuous and increases constantly with a particular slope. The method is based on the concept of the coefficient of subgrade reaction with consideration to soil's behaviour extended to elastic nature. A parametric study is also conducted to study the behaviour of deflection and flexural capacity of piles with respect to modulus of subgrade reaction. The soil is categorized as linear spring as per Winkler's spring model. Various soil types are considered from the studies of Poulos and Davis. Reese and Matlock's non-dimensional analysis procedure is used for the credibility of the study. Finally, by regression, power functions are obtained to establish a relation between depth of zero moment and zero deflection with modulus of subgrade reaction. The full study is conducted using a finite element software ETABS.

Keywords: Foundation, pile, soil, homogenous, lateral load, deflection.

1. Introduction

1.1 General

Numerous research studies, both theoretical and experimental, have been performed on laterally loaded piles for more than six decades. Although extensive theoretical approaches for analyzing the behavior of laterally loaded piles have been developed in recent years, all these methods have limitations. Lateral loading acting on piles involve interaction of soil and water which is considered to be a problem for the deep foundation. So, the solution to this problem requires use of Finite Element Method (FEM) as response of soil is a nonlinear function of deflection of foundation. The Finite Difference Method of solving the differential equation for a laterally loaded pile is very much in use where computer facilities are

available. Reese et al., (1974) and Matlock (1970) has developed the concept of p-y curves for solving laterally loaded pile problems.
To solve problems of laterally loaded single pile, popular analytical approaches developed for lateral load on single pile are:

- The subgrade reaction approach (Matlock, H. and Reese, L.C., 1960), in which continuous nature of the soil is not taken into consideration. Pile reaction is simply related to the deflection of that point.
- The elastic approach (Poulos, 1971), giving a different approach for solving laterally loaded pile problems, based on the theory of elasticity.

Modulus of subgrade reaction is relatively simple and has been used in practice for a long time. This method can incorporate factors such as nonlinearity, variation of subgrade reaction with depth and also can account for various soil layers. As the continuum nature of soil is ignored in subgrade reaction approach, it is considered as a disadvantage of subgrade reaction approach. But in the elastic approach, the soil is assumed to be an ideal elastic continuum. The former method is simple of use. Elastic approach does not take into account the soil yielding and suitable for prediction of load deflection of laterally loaded piles.

This study presents the Modulus of Subgrade Reaction Approach (Reese and Matlock, 1960) assuming the soil as a series of independent linear elastic spring. The advantage of this method is, it can incorporate nonlinearity but by ignoring continuity of soil and layered systems, it suffers from some disadvantages too.

This study is based on the concept of the coefficient of subgrade reaction; and the behavior of soil properties has been extended to include elastic behavior. It has been assumed that the coefficient of subgrade reaction increases linearly with depth for normally consolidated clay and stays constant with depth for an over consolidated clay.

1.2 Scope and objectives

The study presents a simplified and convenient method of determination of maximum moment and maximum deflection of a free headed pile in elastic condition, finding out a generalized equation for determining depth of zero deflection and zero moment on the basis of subgrade reaction. Modulus of subgrade reaction remains constant for over consolidated clay and linear for normally consolidated soil. So, on the basis on n_h and $k_{h,}$ stiffness of the soil is measured and the piles are analyzed on the basis of Winkler's Spring model. The study is expected to be beneficial in analysis of free headed piles embedded in cohesionless and cohesive soil under influence of lateral load implemented by earthquake, wind load, horizontal loads due to movement of equipment mounted on foundations etc.

The study has been done with a view to determine:

- Effect of linear modulus of subgrade reaction in case of cohesionless soil and constant modulus of subgrade reaction for cohesive soil
- Analysis of foundation behavior by extending soil properties to elastic state
- Normalization of moment curve and deflection curve for simplicity of analysis
- Behavior of unrestrained pile under the effect of lateral load with stiffness of homogenous soil
- Determining generalized equation for calculating the depth of zero moment and zero deflection with respect to modulus of subgrade reaction and n_h

2. Method and result for analysis

2.1 General

The moment and deflection at a definite depth of a free headed long pile was determined using parametric analysis software ETABS. Several analyses were undertaken on free headed piles in cohesive and cohesion-less soil under influence of lateral load to predict the deflection and moment at definite depths.

Table 1.1
Non-dimensional method and software analysis for moment of 30ft pile in loose dry sand (n_h=4 ton/ft^3)

Depth (ft)	T	z	A_m	P (kip)	Moment computed from non-dimensional method (kip-ft)	Moment computed using ETABS (kip-ft)
0	7.796	0	0	10	0	0
1	7.796	0.128271	0.11	10	8.5756	9.801
2	7.796	0.256542	0.245	10	19.1002	19.238
3	7.796	0.384813	0.365	10	28.4554	28.014
4	7.796	0.513084	0.459	10	35.78364	35.898
5	7.796	0.641355	0.554	10	43.18984	42.718
6	7.796	0.769625	0.624	10	48.64704	48.362
7	7.796	0.897896	0.692	10	53.94832	52.77
8	7.796	1.026167	0.727	10	56.67692	55.928
9	7.796	1.154438	0.765	10	59.6394	57.86
10	7.796	1.282709	0.769	10	59.95124	58.626
11	7.796	1.41098	0.772	10	60.18512	58.313
12	7.796	1.539251	0.759	10	59.17164	57.026
13	7.796	1.667522	0.743	10	57.92428	54.886
14	7.796	1.795793	0.696	10	54.26016	52.023
15	7.796	1.924064	0.662	10	51.60952	48.572
16	7.796	2.052335	0.615	10	47.9454	44.667
17	7.796	2.180605	0.543	10	42.33228	40.437
18	7.796	2.308876	0.482	10	37.57672	36.006
19	7.796	2.437147	0.431	10	33.60076	31.49
20	7.796	2.565418	0.368	10	28.68928	26.996
21	7.796	2.693689	0.308	10	24.01168	22.618
22	7.796	2.82196	0.238	10	18.55448	18.442
23	7.796	2.950231	0.194	10	15.12424	14.544
24	7.796	3.078502	0.144	10	11.22624	10.991
25	7.796	3.206773	0.109	10	8.49764	7.844
26	7.796	3.335044	0.069	10	5.37924	5.157
27	7.796	3.463315	0.045	10	3.5082	2.98
28	7.796	3.591585	0.015	10	1.1694	1.363
29	7.796	3.719856	0.006	10	0.46776	0.354
30	7.796	3.848127	0.002	10	0.15592	0

2.2 Analysis procedure

- The pile is considered as a cantilever beam with spring portraying as a simulation of soil at definite interval of depths in the ETABS software.
- To get a more accurate result, the pile is divided after each 1ft interval of equal length elements for precise calculation.
- Material property of concrete pile is considered a software default of E_S=518400 ksf
- The spring constant (k_s) of the pile is calculated by multiplying the horizontal modulus of subgrade reaction (k_h) which is constant with depth with the influence area of the pile.

$$ks = kh * A^{i} \quad (1)$$

- For cohesion less soil, the horizontal modulus of subgrade reaction (k_h) is derived from the relationship between stiffness of soil with depth for sand.

$$kh = nh * \left(\frac{z}{D}\right) \tag{2}$$

- The pile is considered free headed, so no support is provided at the pile head.
- In case of every analysis, we have considered the L/D≥10 for each pile.
- The spring constant are put on the horizontal x direction with one spring placed at the tip of the pile on the vertical z direction.
- A lateral load is applied on the tip of the pile and subsequent deflection and moment for the applied load is analyzed.
- After analysis, the values for moment and deflection are achieved from the output table after running the simulation in the ETABS software.

2.3 Comparison between non-dimensional method and software analysis

For free headed piles with, a lateral load of 10 kip is applied at the pile head. In this analysis, we have considered a 30ft pile embedded in loose dry sand (as per Terzaghi).

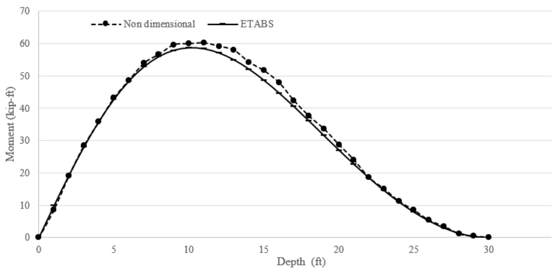

Fig. 1.1. Comparison between Non-dimensional method and Software analysis for moment

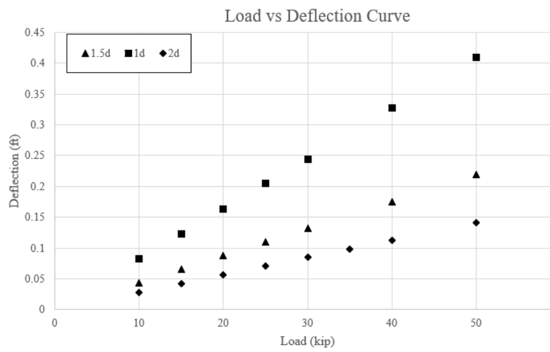

Fig. 1.2. Load-Maximum Deflection curve for Cohesion less soil

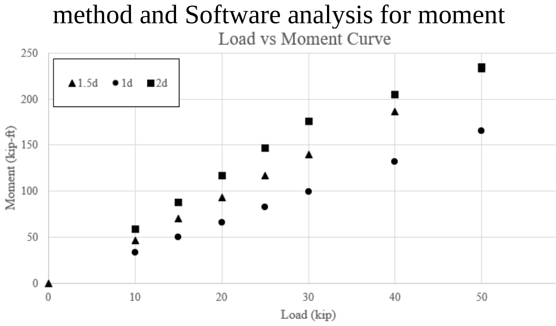

Fig. 1.3. Load-Maximum Moment Curve for Cohesion less soil.

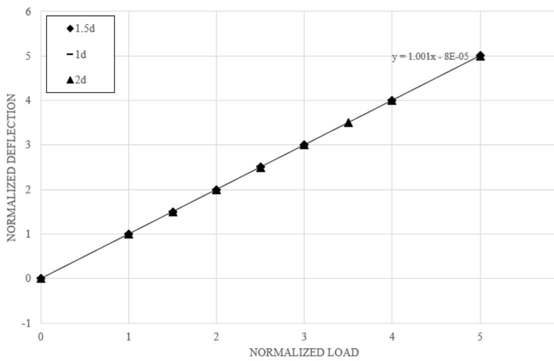

Fig. 1.4. Normalized Load-Normalized Deflection Curve for Cohesion less soil.

2.4 Normalization of moment and deflection for different loads and different pile diameter

Normalization of curve for maximum moment and tip deflection (maximum deflection) is done by performing a dimensionless analysis. While in elastic method the normalization is done by dividing the calculated maximum moment and calculated maximum deflection by the yield moment or yield deflection respectively, we use a particular load's output parameters for determining it in statistic method in our non-elastic analysis in case of cohesion less soil. For piles of a fixed length of 30 ft of different pile diameter, lateral load of different magnitudes was applied at the pile head embedded in loose sand. To obtain the normalized curve for load-maximum moment and load-maximum deflection for a free headed pile embedded in soil with linearly increasing subgrade reaction (cohesion-less soil) subjected to a lateral force, a total of 21 cases were analyzed on ETABS. The analyzed maximum moment vs applied load and maximum defection vs applied load are plotted in Figures 1.2 and 1.3 respectively.

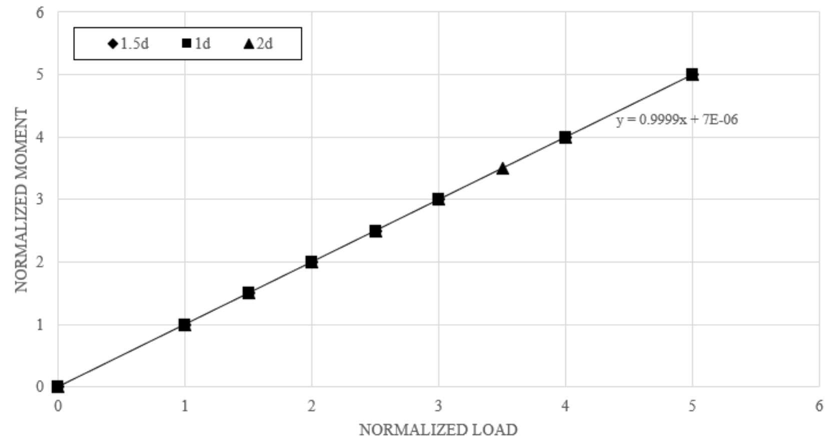

Fig. 1.5. Normalized Load-Normalized Moment Curve for Cohesion less soil

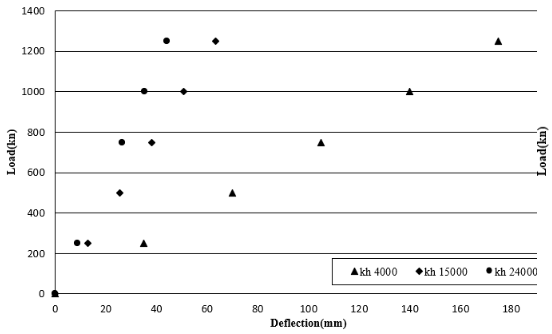

Fig. 2.1. Load-Maximum Deflection curve for Cohesion less soil (Different Kh)

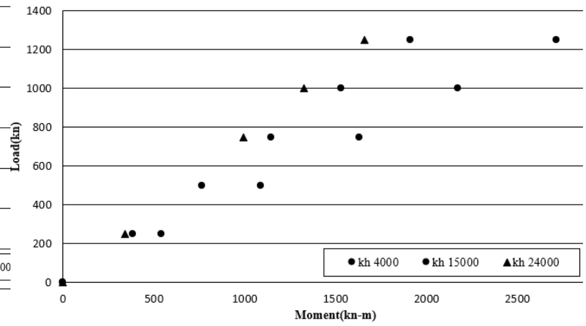

Fig. 2.2. Load-Maximum Moment curve for Cohesion less soil (Different Kh)

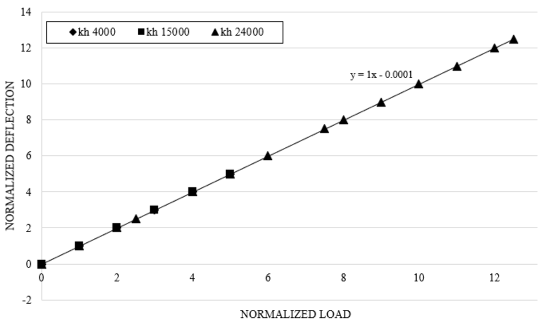

Fig. 2.3. Normalized Load-Normalized Deflection Curve for Cohesion less soil (different kh)

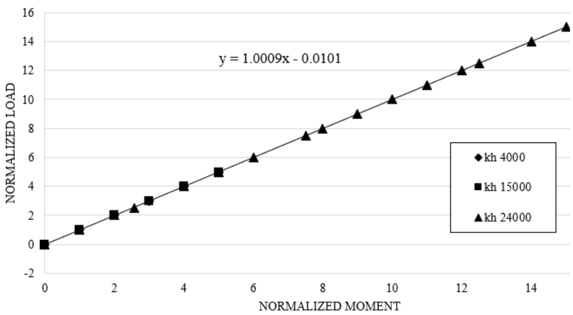

Fig. 2.4. Normalized Load-Normalized Moment Curve for Cohesion less soil (different kh)

Due to the elastic analysis by considering N.C. state of cohesion less soil, we obtain linear curve for both deflection and moment insisting the absence of yield state. It is seen that the all the curves in Figures 1.1 and 1.2 merge into a single curve when the horizontal axis is divided by a particular load and vertical axis is divided by the analyzed moment and deflection of the specific load. These non-dimensional curves show similar shape for different parameters as shown in Figure 1.4 for different diameters of pile and Figure 1.5 for different k_h of soil.

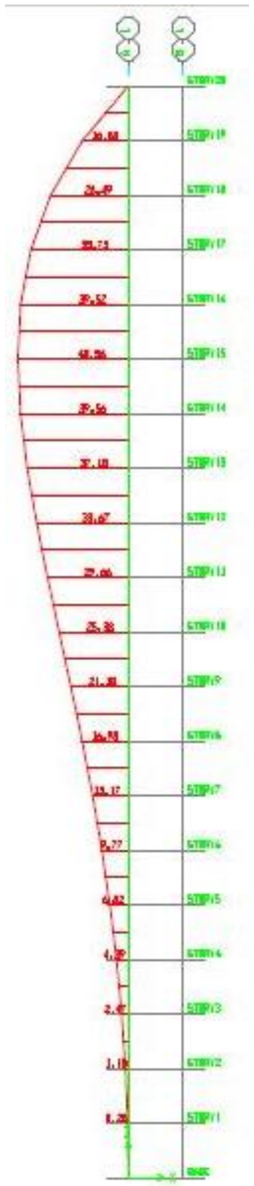

Fig. 3.1. Moment diagram for 40’ pile of dia 1.5’ for kh=4000

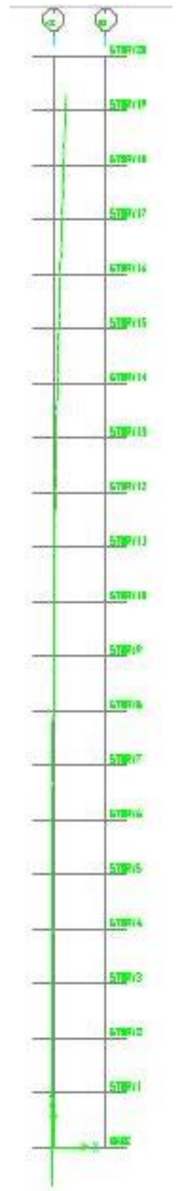

Fig. 3.2. Deformed shape for 40’ pile of dia 1.5’ for kh=4000

Table 1.2
Non-dimensional method and software analysis for deflection of 30ft pile in loose dry sand

Diameter (ft)	E	I	n_h (ton/ft^3)	T	P (kip)	Tip deflection computed by non-dimensional method (inch)	Tip deflection computed by ETABS (inch)
2	518400	0.7854	4	7.796	10	0.339	0.337

Similarly, for different cohesion less soil with different modulus of subgrade reaction, we can normalize the load-moment and load- deflection curve too. (For loose soil, k_h=4000, for medium soil, k_h=15000, for dense soil, k_h=24000). For a 40 ft free headed pile of a constant diameter of 1 ft embedded in different types of soil, different magnitudes of lateral load are applied at the pile head.

2.5.1 Formulation of regression equation

The relation between load-maximum deflection and load-maximum moment can be normalized on the basis of - pile diameter, modulus of subgrade reaction, maximum deflection and maximum moment of piles. Also, the slope of the normalized curve in both cohesion-less and cohesive is a constant of 45^{o} and the straight lines pass through the origin. So, it can be formulated as

For force-maximum deflection $\frac{P}{P^{i}} = \frac{u}{u^{i}}$ *(3)*

For force-maximum-moment $\frac{P}{P^{i}} = \frac{M}{M^{i}}$ *(4)*

These equations can be used for analysis regarding elastic soil where yielding is disregarded hypothetically and straight-line function can be used.

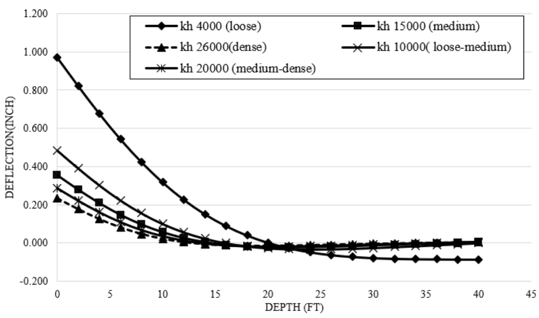

Fig. 4.1. Deflection curve for L=40ft, d=1.5ft pile

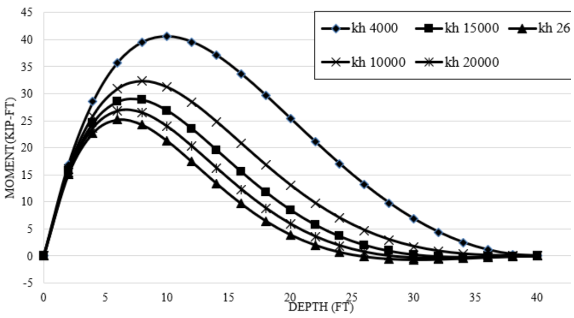

Fig. 4.2. Moment curve for L=40ft, d=1.5ft pile

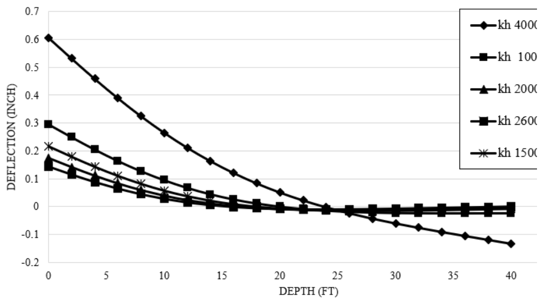

Fig. 4.3. Deflection curve for L=40ft, d=2ft pile

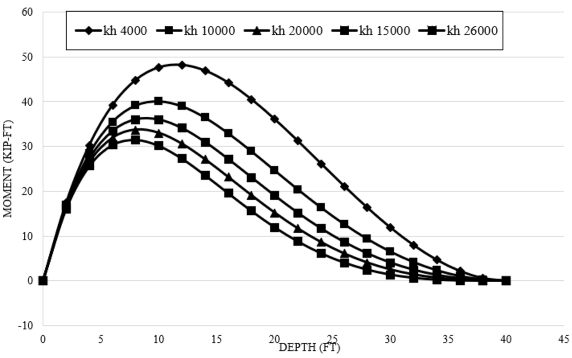

Fig. 4.4. Moment curve for L=40ft, d=2ft pile

2.5.2 Usage of normalized curve

The normalized curve can be used in determining the flexural capacity of pile in simplified way. For each pile, if the normalization factor is known (which in this case is a load for which moment and deflections were already calculated), the flexural capacity and deflection for each pile can be known.

As the normalized curve displays proportional behavior, the maximum moment and maximum deflection can be determined by multiplying the value from the curve by the normalization factor for any load in normally consolidated soil with linear k_h.

2.6 Deflection and moment of long piles

Upon the application of lateral load on a pile, it demonstrates deflection which decreases with depth and moment similar to action on cantilever beams. The modulus of subgrade reaction also increases linearly with depth.

In case of all these analyses, we have assumed free headed piles embedded in over consolidated clay of constant $k_{h.}$ A load of 10 kip is applied in each case. Along with loose, medium and dense soil according to Terzaghi's provision, we assumed a loose-medium soil of k_h =10000 kN/m^3 and medium-dense soil of k_h=20000 kN/m^3.

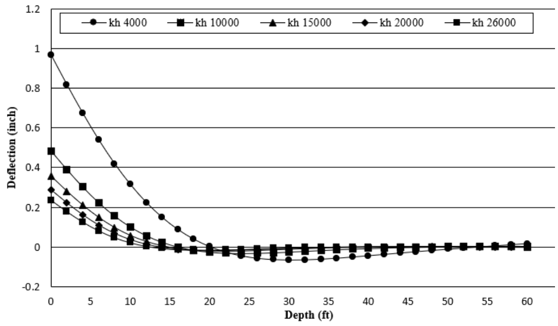

Fig. 4.5. Deflection curve for L=60ft, d=1.5ft pile

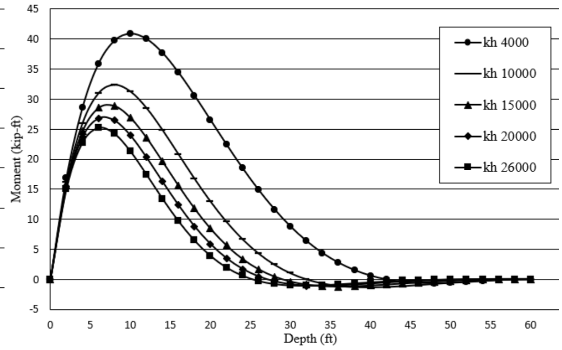

Fig. 4.6. Moment curve for L=60ft, d=1.5ft pile

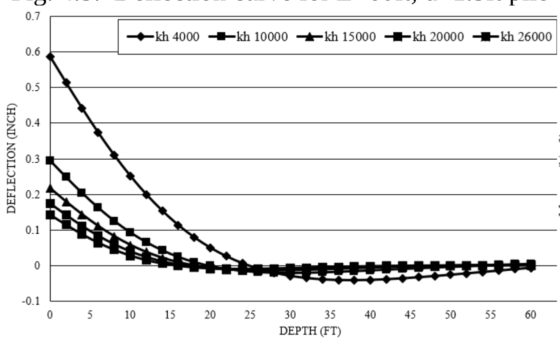

Fig. 4.7. Deflection curve for L=60ft, d=2ft pile

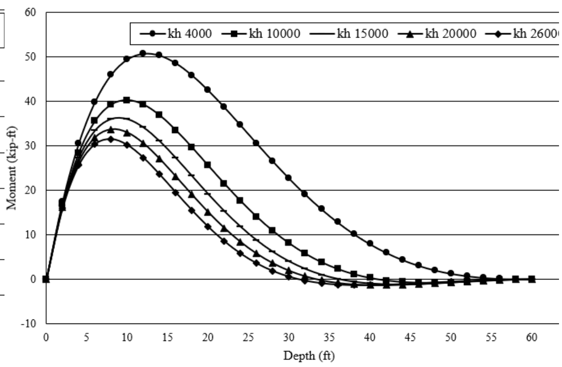

Fig. 4.8. Moment curve for L=60ft, d=2ft pile

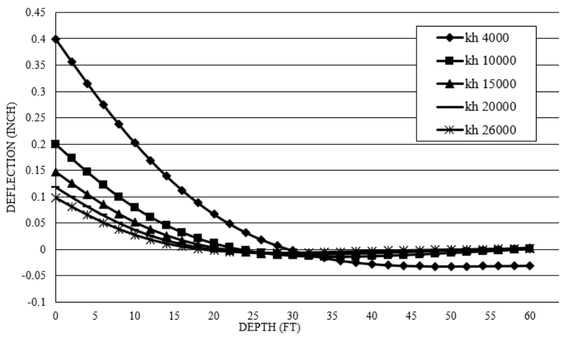

Fig. 4.9. Deflection curve for L=60ft, d=2.5ft pile

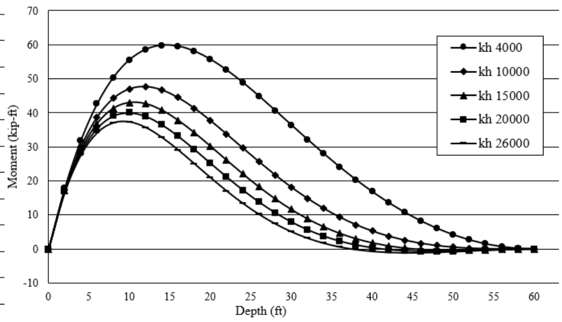

Fig. 4.10. Moment curve for L=60ft, d=2.5ft pile

From the determined values of moment and deflection, a relationship between depth and modulus of subgrade reaction can be established.

The results show that the depth of zero deflection increases with the increase of length of the pile and the difference of this depth decreases with increase of modulus of subgrade reaction as the slope of the trend gets linear. For long piles, after a certain depth, the deflection becomes constant. The depth where zero deflection occurs can be determined in term of modulus of subgrade reaction for a definite diameter of pile.

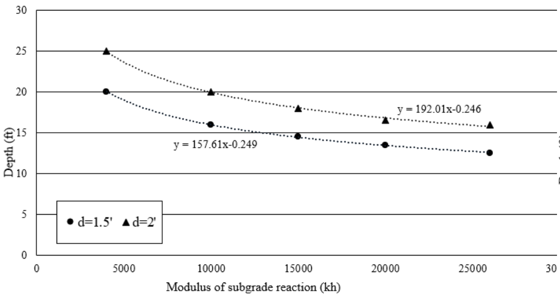

Fig. 5.1. Zero Deflection curve for 40ft pile

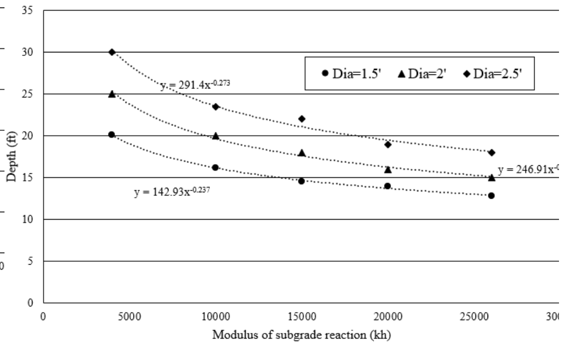

Fig. 5.2. Zero Deflection curve for 60ft pile

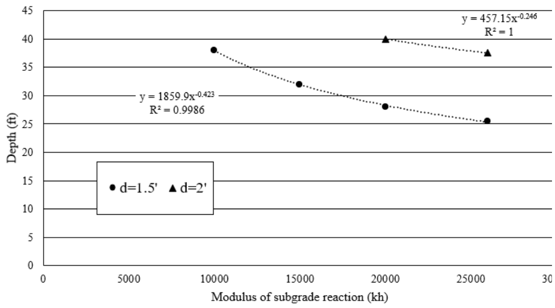

Fig. 5.3. Zero Moment curve for 40ft pile

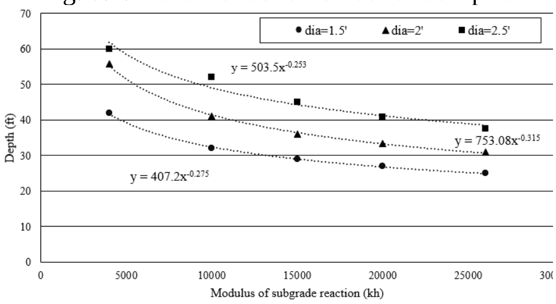

Fig. 5.4. Zero Moment curve for 60ft pile

The results from Figure 5.4 show that the depth of zero moment increases with the increase of length of the pile and the difference of this depth decreases with increase of modulus of subgrade reaction. The depth where zero moment occurs can be determined in term of modulus of subgrade reaction for a definite diameter of pile.

Similarly, a relationship between stiffness of soil (n_h) and deflection can be established for cohesion less soil

2.7 Regression equation

By regression method, it has been found that the power function fit the depth of pile-modulus of subgrade reaction curves. For each analysis, the regression was done to fit linear, hyperbolic, exponential and polynomial function. But for power function, it was obtained that the $R^2 \approx 1$, implying the curves fit this function most.

Relation between zero deflection-k_h $y = Ax^n$ (5)

Where,

y= depth of zero deflection

A= non-dimensional constant

x=k_h of the soil (variable)

n= an exponential constant

From the analysis, we can see that, n= 0.2 ~ 0.3

Again, relation between zero moment-k_h $y = Bx^m$ (6)

y= depth of zero moment

B= non-dimensional constant

x=k_h of the soil (variable)

m= a exponential constant

From the analysis, we can see that, m= 0.2 ~ 0.4

In cohesion less soil, the deflection is stiffness of soil (n_h) dependent, owing to incremental modulus of subgrade reaction with depth as n_h remains constant.

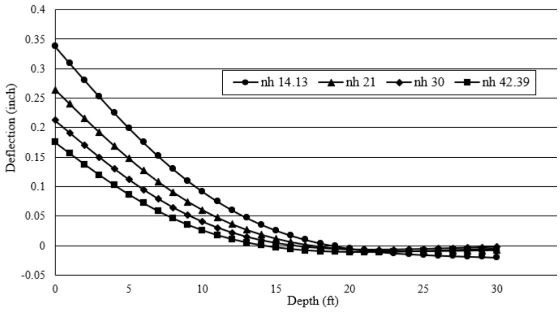

Fig. 6.1. Deflection curve for L=30ft, d=2ft pile (cohesion less soil)

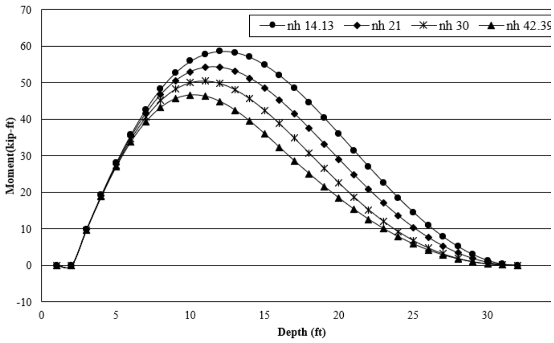

Fig. 6.2. Moment curve for L=30ft, d=2ft pile (cohesion less soil)

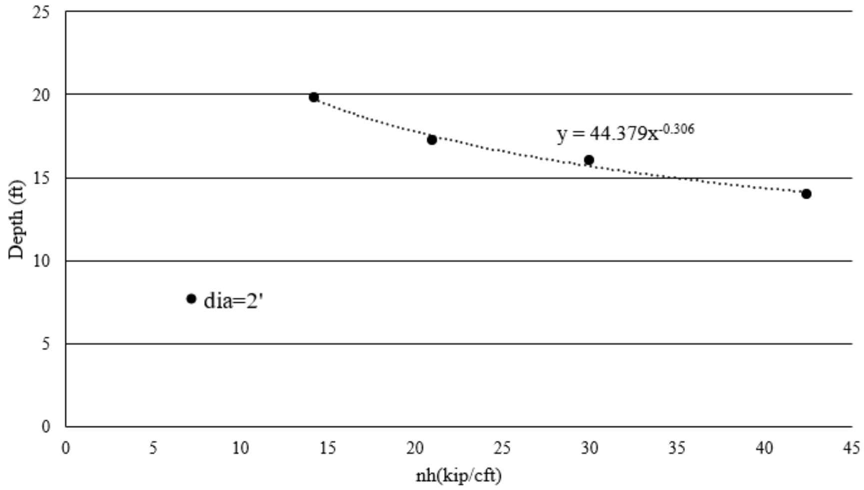

Fig. 6.3. Zero Deflection curve for 30ft pile (for cohesion less soil)

3. Conclusions

The study of this pile foundation is done assuming the soil as elastic disregarding the elastic behavior of pile material to simplify it. In practical cases, totally elastic scenario is impossible, the analysis done here are limited until the pile reaches yield condition. As the soil is regarded to have behaved elastically simplified results are obtained with linear k_h for cohesion less soil and constant k_h for cohesive soil.

The normalization of curves developed a simplified method for analyzing a laterally loaded long pile in uniform soil. While the approximation of depth of zero moment and zero deflection can be done from the regression analysis, which is expected to beneficial in the study of pile foundation design. From this study, we can conclude that:

- The slope of normalized moment curve and normalized deflection curves is a constant of 45° and are a linear function
- The maximum moment and maximum deflection of any piles can be determined from the deflection or moment of known pile diameter and normalization factor, simplifying the determination of flexural capacity of piles.
- For force-maximum deflection the relation becomes $\frac{P}{P'} = \frac{u}{u'}$ and for force-maximum-moment the relation is $\frac{P}{P'} = \frac{M}{M'}$ in the normalized curve

- The maximum deflection always occurs at the pile head and the zero deflection occurs at the near bottom of the pile. So, it is effective to use a short pile in construction.
- Point of zero deflection becomes constant for higher modulus of subgrade reaction. The maximum deflection increases for low modulus of subgrade reaction and decreases for higher modulus of subgrade reaction. So, the analyzed deflection is higher for loose soil and lower for dense soil.
- As the point for zero moment is lower for longer pile, it's preferable to use long pile with less depth to resist major tension crack
- By regression, the determination of depth zero moment with respect to modulus of subgrade reaction is simplified a power function of $y = Bx^m$. Similarly, the determination of depth zero deflection with respect to modulus of subgrade reaction is simplified a power function of $y = Ax^n$
- The slope of the power function for depth of zero deflection-k_h is n=0.2 to 0.3 while for depth of zero moment-k_h, the value is m=0.2 to 0.4

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