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BEHAVIOUR OF SPATIAL FRAME-SHEAR WALL SYSTEMS ON FLEXIBLE BASES

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ABSTRACT: Static linear elastic analysis of spatial frame-shear wall systems on flexible bases, subjected to lateral loading has been considered in this work. The stiffness method was adopted for developing a computer programme, in which the base flexibility has been modelled by linear and rotational springs. The rigid zone end concept has been extended to 3-D structures for modelling the beam-shear wall interface. Typical bisymmetric and monosymmetric spatial frame-shear wall structures have been analysed. The effect of base flexibility on the response of structure e.g. lateral deflection, rotation, angle of twist and bending moment were observed. Bisymmetric spatial frame-shear wall systems have been analysed considering both linear and rotational springs separately and in combination. Monosymmetric structure behaviour has been studied by varying spring stiffnesses, storey height, and moments of inertia of beams for bottom five stories. Deflections and rotational angles are plotted against storey numbers for both the structures considering different base flexibilities. For monosymmetric structure, influence of storey numbers on angle of twist and bending moment are also studied. Deflections and rotations of bisymmetric structures are found to be significantly influenced by flexibility of linear spring systems. Influence of rotational spring is found insignificant. The bending moments at lower levels of shear walls are affected more significantly by base flexibility compared to those at upper levels.

KEY WORDS: Spatial, frame-shear wall, flexible base, rigid zone end, bisymmetric, monosymmetric, stiffness.

INTRODUCTION

The majority of works for the analysis of frame shear wall structures assume rigid foundations for such structures. In practice, however, bases for structures have always some degrees of flexibility. Nevertheless, due to the complexity of the mathematical analysis, engineers use simplified analysis procedures for the design of spatial frame-shear wall structures. Such procedures are usually based on gross approximations, which neglect soil-structure interaction as well as complicated structure behaviour. Depending on the form of structures and the particular soil conditions encountered, it is desirable to estimate the effect of base flexibility on the behaviour of spatial frame-shear wall systems. With this objective, a study on the behaviour of spatial frame-shear wall structures on flexible bases has been carried

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out in this work.

Coull (Coull 1971) reported the first study on the effect of elastic bases on coupled walls. However, few papers have dealt with the three-dimensional soil-structure interaction problems. Swaddiwadhipong (Swaddiwadhipong et al 1986) was the first to investigate three-dimensional flexible foundations using continuous approach method. The discrete force approach to shear wall analysis was employed by Johnson and Choo (Johnson and Choo 1993) for planar walls and by Johnson and Nadjai (Johnson and Nadjai 1993) for spatial shear wall systems. Johnson and Nadjai (Johnson and Nadjai 1994) extended the method to the elasto-plastic condition assuming that plasticity is restricted to the connecting beams. Nadjai and Johnson (Nadjai and Johnson 1996) did further improvement of the method for the elastic and elasto-plastic analysis of planar shear walls on flexible bases. They have shown how the elastic formulation established for coupled shear walls on flexible bases may be extended to three-dimensional shear wall arrangements by discrete force method.

It is intended in this work to study the effect of base flexibility on the behaviour of three-dimensional frame-shear wall systems employing stiffness method. A computer programme is developed for the purpose using rigid zone end concept for beam-shear wall interface and elastic linear and rotational springs to model the base flexibility. Widely used structural analysis software SAP90 is employed to check the reliability of the developed programme.

MODELLING OF FRAME-SHEAR WALL STRUCTURES ON FLEXIBLE BASES

The stiffness of the flexible foundation may be represented by linear and rotational springs with stiffnesses k_v and k_θ respectively at the base of each frame as shown in Fig. 1. It is assumed that the frame-shear wall and supporting soil remain elastic.

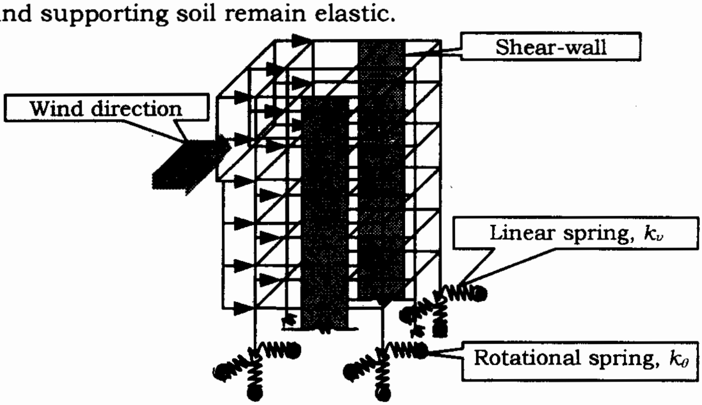


Fig 1. Soil-Structure Interaction Model for a Typical Frame-Shear Wall System

In the spatial frame-shear wall analysis, buildings have been idealised as an assemblage of 3-D frame and planar shear wall systems. Modelling of shear wall in three-dimensional analysis has been done using the concept of rigid zone end conditions at the beam-shear wall interface. The buildings used in the study were bisymmetric and monosymmetric, and rectangular in plan.

Bisymmetric Structures:

A 20-storied structure, symmetric in two mutually perpendicular directions with a storey height of 3.0 m, has been analysed for lateral load in the transverse direction. The plan of the building is shown in Fig. 2. The stiffening elements consist of an arrangement of frames and shear walls, spanning in the x- and z- directions.

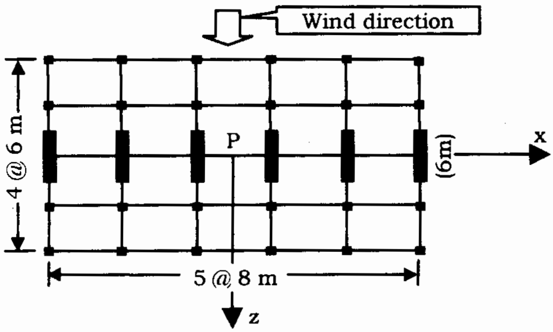


Fig 2. Plan of Bisymmetric Frame-Shear Wall Structural Model

Monosymmetric Structures:

A 15-storied monosymmetric structure, subjected to uniform lateral loading has also been analysed. The structure consists of varying storey heights ranging from 2.75 m to 3.5 m with the plan arrangement shown in Fig. 3. The stiffening elements for the model are frames and shear walls, spanning in the x- and z- direction as shown in figure.

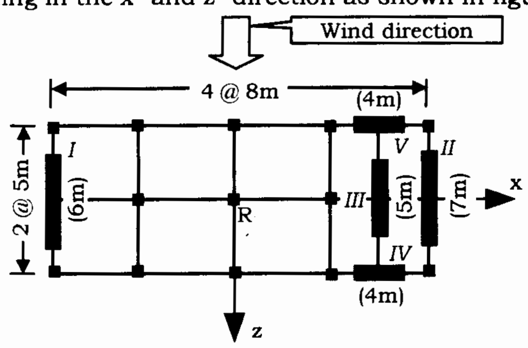


Fig 3. Plan of Monosymmetric Frame-Shear Wall Structural Model

STIFFNESS FORMULATION

Incorporation of Base Flexibilities:

Supports of the structure are modelled by flexible springs, whose stiffnesses have been derived from realistic soil condition. Both linear and rotational springs have been used to model the support stiffness. For a simple two-span beam problem, having elastic support in the mid joint, it can be shown that,

$$\begin{bmatrix} k_{11} + k_v & k_{12} \\ k_{21} & k_{22} + k_\theta \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{1}$$

where, k_v and k_θ are linear (vertical) and rotational stiffness respectively; k_{11}, k_{12}, k_{21}, k_{22} are beam stiffnesses; δ_1, δ_2 are nodal displacements; and P_1, P_2 are joint loads.

Rigid Zone Ends:

Shear walls are usually connected by beams. For purposes of analysis, the stiffness of such connecting beams corresponding to degrees of freedom at the wall axis may be obtained. Ghali and Neville (Ghali and Neville 1987) have described the derivation of modified stiffness matrix for a plane structure. However, for a space structure, the technique to derive the modified stiffness matrix is developed in this work. The "RIGID ZONE END" concept has been used for modification of the stiffness matrix. This concept is illustrated in Fig. 4. There are no member bending and shear deformations within the rigid offset lengths and all member forces are output at the outer ends of the rigid offsets (face of the supports).

In Fig. 4, r_i and r_j are the rigid zone lengths introduced due to the presence of shear wall. Moreover, B is the modified length of the horizontal member that has flexural rigidity. If L is the actual length of the member, then the flexible length B of the member is given by the formula,

$$B = L - (r_i + r_j) \tag{2}$$

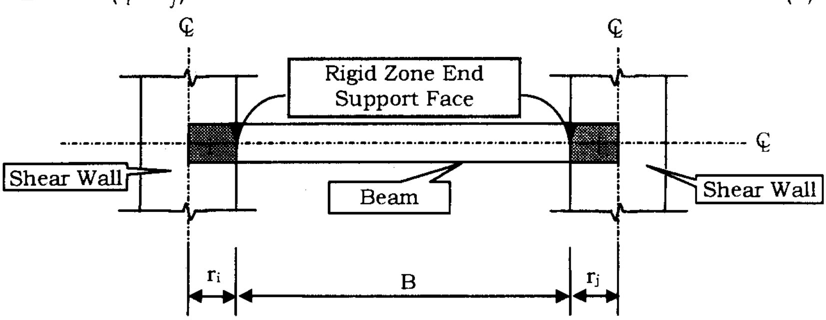


Fig 4. Shear Wall Effect on Beam Geometry

The three-dimensional view of the beam member, with 12 degrees of freedom, is shown in Fig. 5(a). And a sample deflected configuration corresponding to $\delta^*_3=1$ is shown in Fig. 5(b). The beam has two rigid parts SS' and TT' (i.e. r_i and r_j).

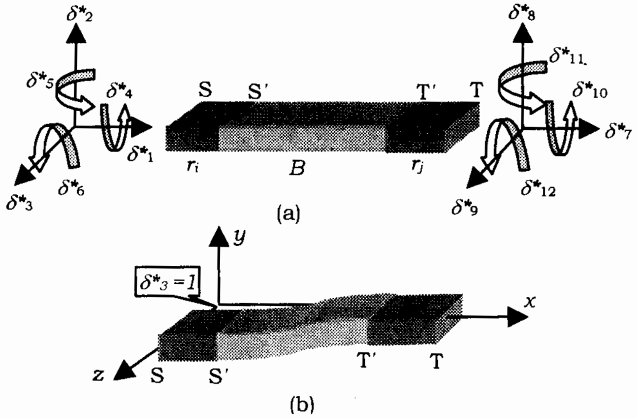


Fig 5. (a) Rigid Ended 3-D Beam Element with 12 Degrees of Freedom, (b) Deflected Configuration of the Element Corresponding to $\delta^*_3=1$.

The displacements $[\delta^*]$ at S and T are related to the displacements $[\delta]$ at S' and T' by geometry as:

$$[\delta] = [H][\delta^*] \tag{3}$$

where, the displacement transformation matrix, $[H]$ is given by:

$$[H] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -r_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & r_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & r_j \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -r_j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The stiffness matrix $[S^*]$ corresponding to displacements $[\delta^*]$ is related to the stiffness matrix $[S]$ corresponding to displacements $[\delta]$ by:

$$[S^*] = [H]^T[S][H] \tag{5}$$

Finally, the stiffness matrix $[S^*]$ corresponding to displacements $[\delta^*]$ can be obtained as:

$$
\left[S^*\right] = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} \tag{6}
$$

Each of the four sub-matrices of $[S^*]$ is a 6x6 square matrix and are given by:

$$
\left[S_{11}\right] = \begin{bmatrix}
\frac{AE}{B} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{B^3} & 0 & 0 & 0 & \frac{6EI_z}{B^2}+\frac{12EI_z r_i}{B^3} \\
0 & 0 & \frac{12EI_y}{B^3} & 0 & -\left(\frac{6EI_y}{L^2}\right) & 0 \\
0 & 0 & 0 & \frac{GJ}{B} & 0 & 0 \\
0 & 0 & -\left(\frac{6EI_y}{B^2}+\frac{12EI_y r_i}{B^3}\right) & 0 & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{B^2}+\frac{12EI_z r_i}{B^3} & 0 & 0 & 0 & \frac{4EI_z}{B}+\frac{12EI_z r_i}{B^2}+\frac{12EI_z r_i^2}{B^3}
\end{bmatrix} \tag{7}
$$

$$
\left[S_{12}\right] = \begin{bmatrix}
-\left(\frac{AE}{B}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & -\left(\frac{12EI_z}{B^3}\right) & 0 & 0 & 0 & -\left(\frac{6EI_z}{B^2}+\frac{12EI_z r_j}{B^3}\right) \\
0 & 0 & -\left(\frac{12EI_y}{B^3}\right) & 0 & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & 0 & -\left(\frac{GJ}{B}\right) & 0 & 0 \\
0 & 0 & \left(\frac{6EI_y}{B^2}+\frac{12EI_y r_i}{B^3}\right) & 0 & \frac{2EI_y}{L} & 0 \\
0 & -\left(\frac{6EI_z}{B^2}+\frac{12EI_z r_i}{B^3}\right) & 0 & 0 & 0 & \frac{2EI_z}{B}+\frac{6EI_z}{B^2}(r_i+r_j)+\frac{12EI_z}{B^3} r_i r_j
\end{bmatrix} \tag{8}
$$

$$
\left[S_{21}\right] = \begin{bmatrix}
-\left(\frac{AE}{B}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & -\left(\frac{12EI_z}{B^3}\right) & 0 & 0 & 0 & -\left(\frac{6EI_z}{B^2}+\frac{12EI_z r_i}{B^3}\right) \\
0 & 0 & -\left(\frac{12EI_y}{B^3}\right) & 0 & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & 0 & -\left(\frac{GJ}{B}\right) & 0 & 0 \\
0 & 0 & -\left(\frac{6EI_y}{B^2}+\frac{12EI_y r_j}{B^3}\right) & 0 & \frac{2EI_y}{L} & 0 \\
0 & \frac{6EI_z}{B^2}+\frac{12EI_z r_j}{B^3} & 0 & 0 & 0 & \frac{2EI_z}{B}+\frac{6EI_z}{B^2}(r_i+r_j)+\frac{12EI_z}{B^3} r_i r_j
\end{bmatrix} \tag{9}
$$

$$
[S_{22}] = \begin{bmatrix}
\frac{AE}{B} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{B^3} & 0 & 0 & 0 & -\left(\frac{6EI_z}{B^2}+\frac{12EI_z r_j}{B^3}\right) \\
0 & 0 & \frac{12EI_y}{B^3} & 0 & -\left(\frac{6EI_y}{L^2}\right) & 0 \\
0 & 0 & 0 & \frac{GJ}{B} & 0 & 0 \\
0 & 0 & \left(\frac{6EI_y}{B^2}+\frac{12EI_y r_j}{B^3}\right) & 0 & \frac{4EI_y}{L} & 0 \\
0 & -\left(\frac{6EI_z}{B^2}+\frac{12EI_z r_j}{B^3}\right) & 0 & 0 & 0 & \frac{4EI_z}{B}+\frac{12EI_z r_j}{B^2}+\frac{12EI_z r_j^2}{B^3}
\end{bmatrix} \quad (10)
$$

where, E is the modulus of elasticity; G is the shear modulus; J is the torsional constant; A is the cross-sectional area of the beam; I_y and I_z are the beam moments of inertia about y- and z- axes respectively.

After the modified stiffnesses for the beam elements connected to shear walls are evaluated, the formulation of the structure stiffness matrix incorporating stiffnesses of other elements is done following standard practice. After assembling the structure stiffness matrix, the stiffnesses of the support springs are added to the diagonal terms of the matrix associated with the degrees of freedom of the support deformations. The solution of the stiffness equations leads to the determination of displacements and member forces. Modified Gauss elimination method has been used for solving for the displacements.

PARAMETRIC STUDY

Different parameters are selected to assess their influence on the behaviour of spatial frame-shear wall systems on flexible bases. As mentioned before, for the purpose of analysis, bisymmetric and monosymmetric model structures have been considered. Parameters that have been varied are:

- base flexibility depending on realistic soil conditions
- beam geometry for bottom five stories
- storey height.

Base Flexibility:

To observe the effect of base flexibility, both linear and rotational springs are taken into consideration. For bisymmetric structure, both the individual and combined effect of the springs are considered, whereas for monosymmetric structure, only combined effect of the springs are considered. The values of different spring constants for different types of soils (Stafford Smith 1991) are shown in Table 1.

Beam Geometry:

To observe the effect of beam strength on deflections and angle of twist, the monosymmetric structure has been taken as the model. The beam moment of inertia of bottom five-stories of the structure has been varied as I, 2I, and 4I, where I represents the beam moment of inertia of the top ten stories.

Table 1. Values of Spring Constants for Different Soils

Type of soil	Linear spring (kN m^{-1})	Rotational spring (kN-m rad^{-1})
Rigid base	∞	∞
Dense sand	7.5e8	1.0e12
Stiff clay	1.2e7	1.0e10
Weak soil	3.0e6	1.0e9
Poor soil	3.0e5	5.0e8
Very poor soil	3.0e3	2.5e8

Storey Height:

The monosymmetric structure has been analysed for four different storey heights to observe the effect on angle of twist and bending moment. The chosen storey heights were 2.75 m, 3.0 m, 3.25 m, and 3.5 m.

INFLUENCE OF DIFFERENT PARAMETERS

Effect of Base Flexibility:

As shear walls are sensitive to the flexibilities of the foundations, both bisymmetric and monosymmetric model structures have been analysed with different base flexibilities.

(i) Effects on bisymmetric model:

Effect of base flexibility on the behaviour of 20-storied bisymmetric structure considering both linear and rotational springs at a time, is shown in Fig. 6. Lateral deflection at different storey levels at point *P* of the plan (Fig. 2) is shown in Fig. 6(a). The corresponding rotations are illustrated in Fig. 6(b). Six different base conditions are taken into account. It is observed that dense sand, stiff clay and weak soil behave like a rigid base. When very poor soil is encountered, the deflection may increase up to 45% as compared to a rigid base. The values of top deflections and top and maximum rotations for different soil types are given in Tables 2 to 4. In these tables, linear and rotational springs are considered independently and in combination respectively. It is observed that, rotational springs do not have practically any effect on deflection and rotation. Linear springs affect the parameters when poor soils are encountered.

Table 2. Deflection and Rotations for 20-Storied Bisymmetric Structure for Different Bases when Linear Springs are Considered

Type of soil	Linear spring (kN m^{-1})	Top deflection (mm)	Top rotation (x10^{-4} rad)	Max. rotation (x10^{-4} rad)
Rigid base	∝	53.2850	11.5	133.7
Dense sand	7.5E8	53.2854	11.5	133.7
Stiff clay	1.2E7	53.3400	11.6	133.8
Weak soil	3.0E6	53.5060	11.8	134.1
Poor soil	3.0E5	55.4840	14.9	137.2
Very poor soil	3.0E3	74.7750	46.2	168.7

Table 3. Deflection and Rotations for 20-Storied Bisymmetric Structure for Different Bases when Rotational Springs are Considered

Type of soil	Rotational spring (kN-m rad^{-1})	Top deflection (mm)	Top rotation (x10^{-4} rad)	Max. rotation (x10^{-4} rad)
Rigid base	∝	53.2850	11.50	133.70
Dense sand	1.0E12	53.2845	11.48	133.72
Stiff clay	1.0E10	53.2846	11.48	133.72
Weak soil	1.0E09	53.2847	11.50	133.72
Poor soil	5.0E08	53.2848	11.48	133.72
Very poor soil	2.5E08	53.2851	11.48	133.72

Table 4. Deflection and Rotations for 20-Storied Bisymmetric Structure for Different Bases when Both Linear and Rotational Springs are Considered

Type of soil	Linear spring (kN m^{-1})	Rotational spring (kN-m rad^{-1})	Top deflection (mm)	Top rotation (x10^{-4} rad)	Max. rotation (x10^{-4} rad)
Rigid base	∝	∝	53.2850	11.50	133.70
Dense sand	7.5E8	1.0E12	53.2854	11.48	133.72
Stiff clay	1.2E7	1.0E10	53.3398	11.57	133.81
Weak soil	3.0E6	1.0E09	53.5058	11.82	134.07
Poor soil	3.0E5	5.0E08	55.5286	11.12	137.24
Very poor soil	3.0E3	2.5E08	74.7760	46.19	168.66

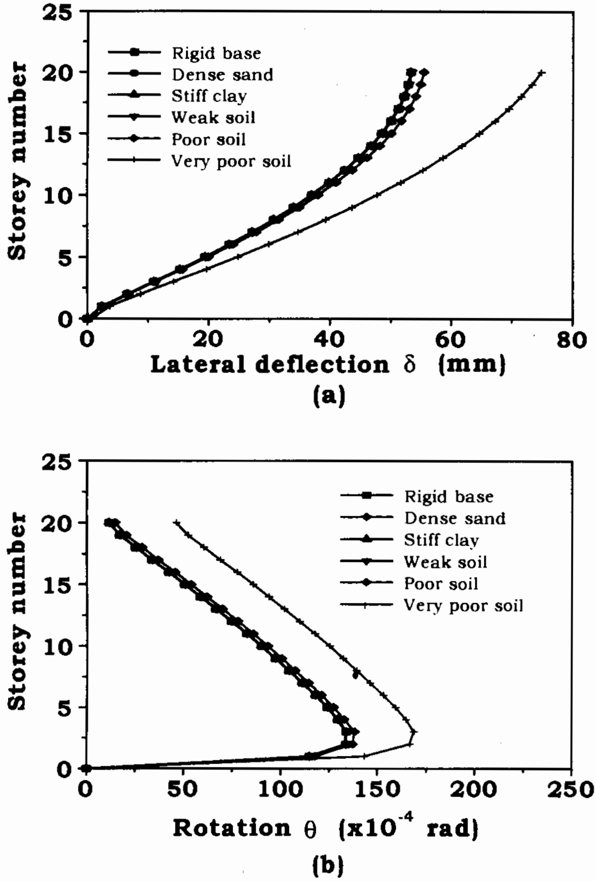
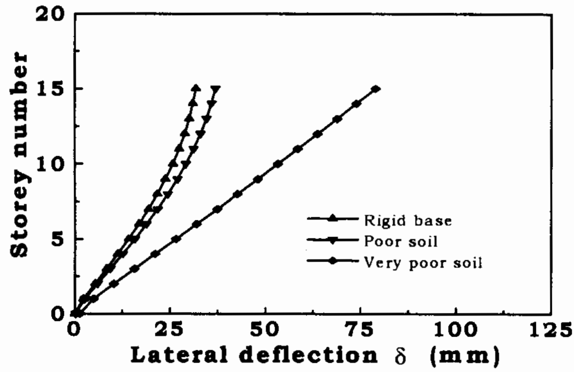


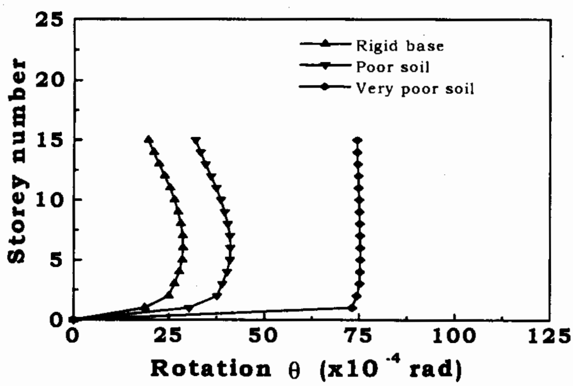
Fig 6. Effect of Foundation Flexibility on the Behaviour of 20-Storied Bisymmetric Structure Considering Both Linear and Rotational Springs at the Base (a) Lateral Deflection at Different Storey Levels (b) Angle of Rotation at Different Storey Levels

(ii) Effects on monosymmetric model:

The distribution of lateral deflection and the angle of rotation at different storey levels at point *R* of the plan (Fig. 3) of the 15-storied monosymmetric structure for three different base conditions are shown in Fig. 7(a) and 7(b) respectively. The effect of rigid base, poor and very poor soils are considered. It may be seen that the deflection and rotation increase significantly as the foundation stiffness decreases. Rigid base assumption in case of poor soils may be highly detrimental, as the deflection may be as high as two and half times compared to that for rigid base. Foundation flexibility affects the overall behaviour of the monosymmetric structure more prominently compared to that of bisymmetric one.



(a)

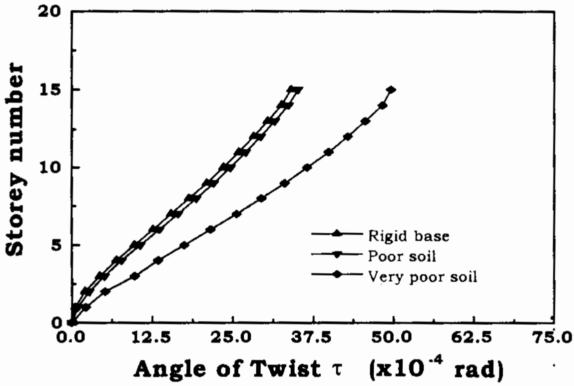


(b)

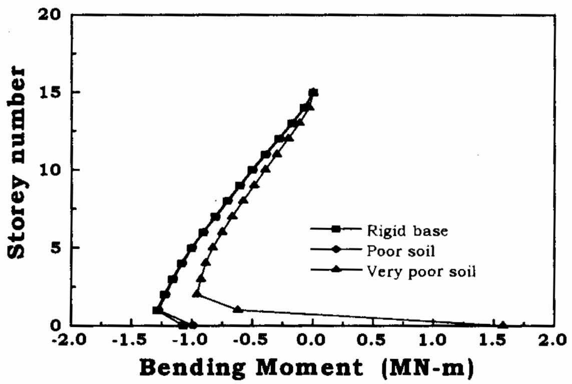
Fig 7. Effect of Foundation Flexibility on the Behaviour of 15-Storied Monosymmetric Structure Considering Both Linear and Rotational Springs at the Base (a) Lateral Deflection at Different Storey Levels (b) Angle of Rotation at Different Storey Levels

As the structure considered is asymmetric in plan, the lateral load would result in both bending and torsional effects. Therefore, the distributions of angle of twist and bending moment of wall I (Fig. 3) at different storey levels are shown in Fig. 8(a) and 8(b) respectively. It can be seen that the angle of twist changes slope over the height. At the bottom portion of the wall, the slope is decreasing in nature, but the slope changes direction as the height increases. This occurs because the torsional stiffness close to the base is governed by the wall system, whereas the frame system predominates higher up the structure. Fig.

8(b) shows that the bending moment at lower levels are more significantly influenced by soil condition than at the upper levels. Angle of twist, on the other hand, is influenced more significantly at the upper levels by soil condition than at the lower levels.



(a)

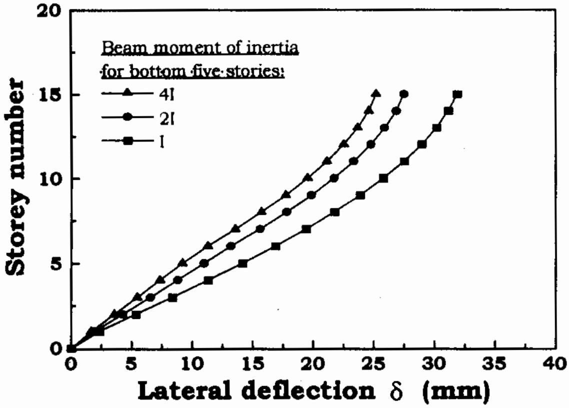


(b)

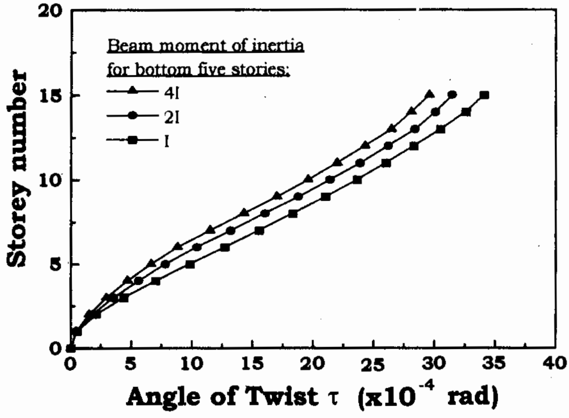
Fig 8. Effect of Foundation Flexibility on the Response of Shear Wall I of the 15-Storied Monosymmetric Structure Considering Both Linear and Rotational Springs at the Base (a) Angle of Twist at Different Storey Levels (b) Bending Moment at Different Storey Levels

Effect of Beam Geometry:

The general idea is that, if the beam moment of inertia of the structure had been increased, then the deflection and angle of twist would decrease. The effect of beam moment of inertia on the behaviour of 15-storied monosymmetric structure is reflected in Fig. 9 for three different cases of beam moment of inertia at bottom five stories viz. I, 2I and 4I. Here, I represents the moment of inertia of beams at upper ten floors. The variations of lateral deflection and angle of twist at different storey levels at point *R* of the plan of the structure are shown in Fig. 9(a) and 9(b) respectively. It can be observed that both the lateral deflection and angle of twist over the full height of the structure reduce significantly with increasing moment of inertia of beams at lower five stories only.



(a)



(b)

Fig 9. Effect of Beam Moment of Inertia on the Behaviour of 15-Storied Monosymmetric Structure on Stiff Clay (a) Lateral Deflection at Different Storey Levels (b) Angle of Twist at Different Storey Levels

Effect of Storey Height:

Behaviour of 15-storied monosymmetric structure is studied in Fig. 10 for varying storey heights such as 2.75 m, 3.0 m, 3.25 m, and 3.5 m. The variations of angle of twist and bending moment at different storey levels at point R of the plan of the structure are shown in Fig. 10(a) and 10(b) respectively.

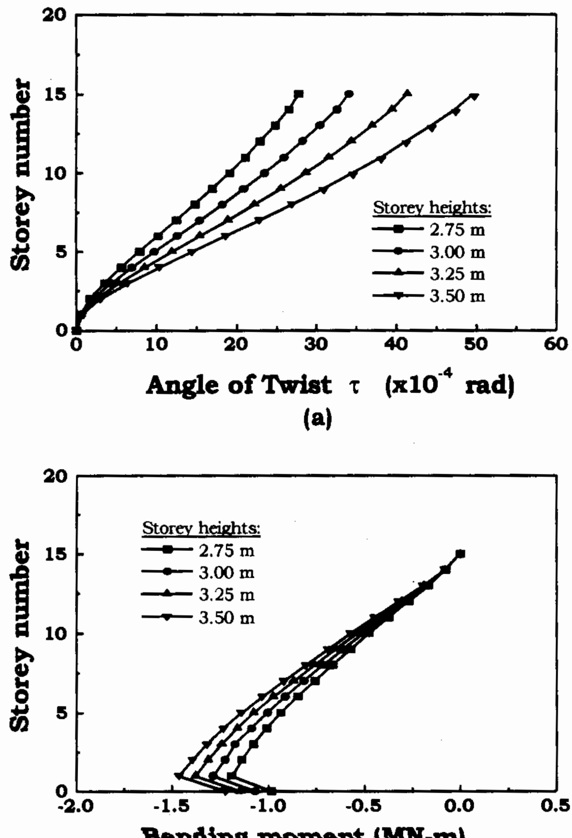


Fig 10. Effect of Storey Height on the Behaviour of 15-Storied Monosymmetric Structure on Stiff Clay (a) Angle of Twist at Different Storey Levels (b) Bending Moment at Different Storey Levels

The height between the floors remains constant over the full height. Angle of twist is influenced more prominently by storey heights at the upper levels (Fig. 10a) whereas, bending moment is influenced more significantly at lower levels by storey heights (Fig. 10b).

CONCLUSIONS

From the investigations carried out, the following conclusions are drawn:

The lateral deflections and rotations of bisymmetric structure are influenced significantly by poor soils represented by linear spring system. Rotational spring stiffnesses, however, have been found to have insignificant effect on the structure response.

Practically, weak soil, stiff clay, and dense sand may be considered as rigid base without affecting the resulting structure response in case of bisymmetric structure. However, poor and very poor soils have to be taken care of in the analysis, as response may be significantly influenced.

For monosymmetric structure, both rotations and deflections of the structure are found to be significantly affected by condition of base flexibility. This effect is more prominent for monosymmetric structure than for bisymmetric.

Slope for angle of twist of shear walls changes direction over the height, which signifies that the torsional stiffness close to the base is governed by the wall system, whereas the frame system predominates higher up the structure.

Both angle of twist and lateral deflection over the full height of a monosymmetric structure reduce significantly with increased moment of inertia of beams at bottom few stories only.

Bending moment at lower stories and the angle of twist at upper stories are more sensitive to storey heights.

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